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Identification of Sources of Variation in Poverty Outcomes

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Abstract

The international community has declared poverty reduction one of the fundamental objectives of development, and therefore a metric for assessing the effectiveness of development interventions. This creates the need for a sound understanding of the fundamental factors that account for observed variations in poverty outcomes either over time or across space. Consistent with the view that such an understanding entails deeper micro empirical work on growth and distributional change, this paper reviews existing decomposition methods that can be used to identify sources of variation in poverty. The maintained hypothesis is that the living standard of an individual is a pay-off from her participation in the life of society. In that sense, individual outcomes depend on endowments, behavior and the *circumstances* that determine the returns to those endowments in any social transaction. To identify the contribution of each of these factors to changes in poverty, the *statistical* and *structural* methods reviewed in this paper all rely on the notion of *ceteris paribus* variation. This entails the comparison of an observed outcome distribution to a counterfactual obtained by changing one factor at a time while holding all the other factors constant.

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Identification of Sources of Variation in Poverty Outcomes

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1. Introduction

Poverty reduction is one of the key objectives of socioeconomic development. The first World Development Report (WDR) argued that development efforts should be aimed at the twin objectives of rapid growth and poverty reduction[1] (World Bank 1978). This vision of development has been reiterated in one form or another in subsequent reports culminating in a conception of development as *opportunity equalization* presented in WDR 2006 (World Bank 2005). In this context, equity is defined in terms of a level playing field where individuals have equal opportunities to pursue freely chosen life plans and are spared from extreme deprivation in outcomes. In this sense, the pursuit of equity also entails that of poverty reduction.

A recent review of poverty trends across the world has shown that poverty had been on a steady decline for a wide variety of countries from the late 1990s up until 2009 (when the financial crisis hit the world economy). The evidence on inequality reduction, a key determinant of poverty outcomes, is however mixed. From the policymaking perspective, it is important to understand the factors driving these observed outcomes.

Focusing on the fact that distributional statistics are computed on the basis of a distribution of the living standards which is fully characterized by its mean and the degree of inequality, several authors have proposed counterfactual decomposition methods to identify the contribution of changes in the *mean* and in *inequality* to variations in overall poverty. These decompositions include the Datt-Ravallion (1992) method, which splits the change in poverty into distribution-neutral growth effect, a redistributive effect and a residual interpreted as an interaction term. The Shapley method proposed by Shorrocks (1999) is analogous to that of Datt and Ravallion, but does not involve a residual. Kakwani (2000) has proposed an equivalent approach. Ravallion and Huppi (1991) offer a way of decomposing change in poverty over time into intrasectoral effects, a component due to population shifts and an interaction term between sectoral changes and population shifts. We present a detailed review of all these macro methods in the appendix.

[1] This recommendation is consistent with the theme underlying the study of redistribution with growth by Chenery et al. (1974). This study advocates the use of explicit social objectives as a basis for choosing development policies and programs. In particular, any development intervention must be evaluated in terms of the benefits it provides to different socio-economic groups.

However, the usefulness of the above described decomposition methods in policymaking is severely limited by the fact that they explain changes in poverty on the basis of changes in summary statistics that are hard to target with policy instruments. The difficulty stems from the fact that such statistics hide more than they reveal about the heterogeneity of impacts underlying aggregate outcomes. It is well known that heterogeneity of interests and of individual circumstances plays a central role in both policymaking and in the determination of the welfare impact of policy. Ravallion (2001) argues that understanding this heterogeneity is crucial for the design and implementation of targeted interventions that might enhance the effectiveness of growth-oriented policies. He further adds that such an understanding must stem from a deeper micro empirical work on growth and distributional change.

The purpose of this paper is to review the essence of existing methods that can be used to identify key factors that drive changes in the observed poverty outcomes. The paper is akin to the excellent review by Ferreira (2010) of the evolution of the methodology for understanding the determinants of the relationship between economic growth, change in inequality and change in poverty. While that review covers the macro-, meso- and micro-economic approaches, we focus on a variety of micro-decomposition methods and delve deeper into the identification strategy underlying each of these methods. The point of departure of these methods is the same as that of the macro methods noted above and presented in the appendix. They too start from the fact that poverty measures, along with many other distributional statistics, can be viewed as real-valued functionals of the relevant distributions[2] so that changes in poverty are due to changes in the underlying distribution of living standards. Macro-decomposition methods proceed by characterizing changes in the underlying distribution in terms of changes in aggregate statistics such as the mean, relative inequality, sub-group population shares and within-group poverty. The micro-decomposition methods reviewed here go beyond these summary statistics and attempt to link distributional changes to fundamental elements that drive these changes.

[2] Roughly speaking, a functional is a function of a function. In this particular context, it is a rule that maps every distribution in its domain into a real number (Wilcox 2005).

The outline of the paper is as follows. Section 2 presents the basic framework underlying all decomposition methods considered in this paper. The logic underpinning all these methods can be organized around the following terms: (i) domain, (ii) outcome model, (iii) scope, (iv) identification, and (v) estimation. The type of distributional change a method seeks to decompose on the basis of a model that links the outcome of interest to its determining factors defines the *domain* of that method. The specification of the *outcome model* associated with a decomposition method determines the potential *scope* of the method, where scope represents the set of explanatory factors the method tries to uncover by decomposition. In other words, the scope defines the terms of the decomposition. Identification concerns the assumptions needed to recover, in a meaningful way, various terms of the decomposition. The outcome model is used to construct *counterfactuals* on the basis of *ceteris paribus* variations of the determinants in order to identify the contribution of each such factor to observed changes in the object of interest. Finally, estimation involves the computation of identified parameters on the basis of sample data. These ideas, which constitute in fact the methodological bedrock of impact evaluation, will be illustrated within the basic Oaxaca-Blinder framework for decomposing changes in the mean of a distribution, and its generalization to any distributional statistic.

Section 3 reviews methods used to identify and estimate the *endowment* and *price effects* along the entire outcome distribution. Decomposing changes in whole distributions of outcomes is bound to reveal heterogeneity in the impact of the growth or development process on economic welfare. Furthermore, a poverty-focused analysis requires an understanding of what goes on at and below the poverty line. This section will therefore focus on the decomposition of differences in density functions and across quantiles based on purely statistical methods that rely on conditional outcome distributions. It discusses the *residual imputation method* proposed by Juhn, Murphy and Pierce (1993) to split the price effect into a component due uniquely to observable characteristics and another due to unobservables. One particular advantage of the statistical approach is that it provides the analyst with semi- and non-parametric methods for the identification of the aggregate endowment and price effects without having to impose a functional form on the relationship between the outcome and its determinants. However, the statistical framework is unable to shed light on the mechanism underlying that relationship. The decomposition results therefore do not have any causal interpretation.

Bourguignon and Ferreira (2005) note that, in addition to the endowment and price effects, long-run changes in the distribution of the living standards are driven by changes in agents' behavior with respect to labor supply, consumption patterns, or fertility choice. A key limitation of the methods reviewed in section 3 is that they fail to account for the effect of behavioral changes in addition to the composition and structural effects. Because these methods are based on statistical models of conditional distributions, it is conceivable that the behavioral effect is mixed up with the price effect identified by these methods. Section 4 therefore considers methods that have been proposed to account for behavioral responses to changes in the socioeconomic environment. All these methods rely on the specification and estimation of a microeconometric model based on some theory of individual (or household) behavior and social interaction. These methods go a step further in trying to identify factors associated with structural elements that underpin observed changes in poverty outcomes. Both the statistical and structural approaches seek to model conditional outcome distributions. A key distinction between the two approaches is that the former relies entirely on statistics while the later combines economics and statistics.

While the methods reviewed here have been applied mostly in labor economics to decompose wage distributions, this review will pay special attention to their adaptability to the decomposition of household consumption which is the basis of poverty measurement. Consideration will also be given to model specifications that drop the assumption of perfectly competitive markets to accommodate situations found most frequently in rural areas in developing countries. Concluding remarks are made in section 5.

2. The Basic Framework

This section presents a theory of *counterfactual decomposition* of variations in individual and social outcomes and illustrates that conceptual framework in the context of the classic Oaxaca-Blinder method and its generalization to the case of a generic distributional statistic.

2.1. A Theory of Counterfactual Decomposition

All decomposition methods considered in this paper (including the macro methods presented in the appendix) are governed by a basic theory of counterfactual decomposition. Each method can be characterized in terms of the following elements: domain, outcome model, scope, identification and estimation procedure. The *domain* is the type of distributional changes the method seeks to decompose (e.g. changes in poverty over time or across space). The *outcome model* links the outcome of interest to its determining factors. A poverty measure, for instance, is a social outcome that is a functional of the underlying distribution of individual outcomes. As noted in the introduction and demonstrated in the appendix, macro-decomposition methods use this fact to link variations in poverty to changes in the mean and relative inequality characterizing the underlying distribution.

Outcome models used by micro-decomposition methods can be motivated as follows. Poverty measurement is based on a distribution of living standards. The living standard of an individual is an outcome of an interaction between opportunities offered by society and the ability of the individual to identify and exploit such opportunities. In other words, the living standard of an individual is a pay-off from her participation in the life of society. One can thus think of life in society as a game defined by a set of rules governing various interactions of the parties involved (players). These rules spell out what the concerned parties are allowed to do and how these allowable actions determine outcomes. An environment within which a game is played consists of three basic elements: (1) a set of potential participants, (2) a set of possible outcomes and (3) a set of possible types of participants (players). Types are characterized by their preferences, capabilities, information and beliefs (Milgrom 2004). The operation of a game can be represented by a function mapping environments to potential outcomes. Thus an individual pay-off is a function of *participation* and *type*. This paradigm motivates our thinking of the living standard of an individual as a function of *endowments, behavior* and the *circumstances* that determine the returns to these endowments from any social transaction.

The outcome model can take the form of a single equation or a set of equations (as we will see in section 4) and ultimately establishes a relationship between the domain and the *scope* of the decomposition method. The *scope* is the set of explanatory factors the method tries to uncover by decomposition. In fact, the specification of the outcome model determines the potential scope of the corresponding decomposition method. For instance, the scope of the macro methods reviewed in the appendix is limited to some aggregate statistics based on the underlying outcome distribution. Such statistics include: the mean, measures of relative inequality, population shares and within-group poverty. The outcome model underlying micro-decomposition methods implies that the potential scope for these methods includes endowment and price effects, and behavioral responses. However, the statistical methods discussed in section 3 are unable to account for behavior because they are based only on the joint distribution of the outcome and individual characteristics. Methods reviewed in section 4 can account for behavioral responses in addition to the endowment and price effects. This ability stems from the fact that these methods combine economic theory of behavior and social interaction with statistics to explain observed outcomes. Behavior, endowments and prices thus represent some of the deep structural elements that drive the distributional changes underlying observed variations in poverty outcomes.

Identification concerns the assumptions needed to recover the factors of interest at the population level. While macro- and micro-decomposition methods differ in their scope (meaning the elements they try to identify) they share the same fundamental *identification strategy* based on the notion of *ceteris paribus* variation. Attribution of outcomes to policy is the hallmark of policy impact evaluation. Indeed, variations in individual outcomes associated with the implementation of a policy are not necessarily due to the policy in question. These variations could be driven by changes in *confounding* factors in the socioeconomic environment. At the most fundamental level, all identification strategies seek to isolate an independent source of variation in policy and link it to the outcome of interest to ascertain impact. Macro and micro methods base identification of the determinants of differences across distributions of living standards on a comparison of counterfactual distributions with the observed ones. Counterfactual distributions are obtained by changing one determining factor at a time while holding all the other factors fixed (this is a straight application of the notion of *ceteris paribus* variation).

Estimation involves the computation of the relevant parameters on the basis of sample data. The linchpin of the whole process is the estimation of credible counterfactuals. In the context of micro-decomposition methods, there is a key counterfactual that must be carefully estimated, namely: the distribution of outcomes in the base state (t=0) assuming the distribution of individual characteristics prevailing in the end state (t=1). Put another way, that counterfactual represents the distribution of outcomes that would prevail in the end state if the characteristics in that state had been treated according to the outcome structure prevailing in the initial state. Depending on the chosen functional form for the outcome equation, there are both parametric and nonparametric ways of estimating this counterfactual. We now consider the translation of these basic ideas into the Oaxaca-Blinder decomposition framework.

2.2. The Classic Oaxaca-Blinder Method

Structure

As discussed in the next subsection, micro-decomposition methods encountered in the literature may be considered a generalization of the classic Oaxaca-Blinder decomposition method. This approach assumes that the outcome variable y is a *linear* function of individual characteristics that is also *separable* in observable covariates x and unobservable factors ε. In addition, it is assumed that the conditional mean of ε given the observables is equal to zero. Focusing on variations over time, we let t=0 for the initial period and 1 for the end period. Then the relationship between y and its determinants can be written as follows.

$$y_t = x_t\beta_t + \epsilon_t; \;\; t = 0, 1. \qquad (1)$$

Abstracting from the time subscript, the conditional mean outcome can be written as follows: $E(y|x) = x\beta$. Therefore β is a measure of the effect of x on the conditional mean outcome. Furthermore, the law of iterated expectations implies that the unconditional mean outcome is:$E(y) = E_x[E(y|x)] = E(x)\beta$. This result implies that β also measures the effect of changing the mean value of x on the unconditional mean value of y. This is the interpretation underlying the original Oaxaca-Blinder decomposition (Fortin, Lemieux and Firpo 2011).

Let $\Delta_O^{\mu} = [E(y_1) - E(y_0)]$ represent the overall difference in unconditional mean outcome between the two periods. This is the domain of the classic Oaxaca-Blinder method. This domain can also be expressed as: $\Delta_O^{\mu} = [E(x_1)\beta_1 - E(x_0)\beta_0]$. The average outcome for period 1 valued on the basis of the parameters for period 0 is equal to $E(x_1)\beta_0$. This is a counterfactual outcome for period 1. We can subtract it from and add it back to the above overall mean difference to get the following expression[3].

$$\Delta_O^{\mu} = [E(x_1) - E(x_0)]\beta_0 + [E(x_1)(\beta_1 - \beta_0)] \qquad (2)$$

Looking at the regression coefficients β as characterizing the returns to (or reward for) observables characteristics, this *aggregate decomposition* reveals that, under the maintained assumptions (i.e. identifying assumptions), the overall mean difference can be expressed as: $\Delta_O^{\mu} = \Delta_X^{\mu} + \Delta_S^{\mu}$, where Δ_X^{μ} is the *endowment* effect and Δ_S^{μ} is the *price* effect[4].

The assumption of additive linearity implies that one can also perform a *detailed decomposition* whereby the endowment and price effects are each divided into the respective contribution of each covariate. To see this formally, let x_k and β_k stand respectively for the k^{th} element of x and β. Then the endowment and price effects can be written in terms of sums over the explanatory variables. For the endowment effect, we have

$$\Delta_X^{\mu} = \sum_{k=1}^{m}[E(x_k|t=1) - E(x_k|t=0)]\beta_{0k} \qquad (3)$$

Similarly for the structural effect, we have the following expression

$$\Delta_S^{\mu} = \sum_{k=1}^{m} E(x_k|t=1)(\beta_{1k} - \beta_{0k}) \qquad (4)$$

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[3] An alternative expression is based on this counterfactual: $E(x_0)\beta_1$. The corresponding decomposition is: $\Delta_O^{\mu} = [E(x_1) - E(x_0)]\beta_1 + E(x_0)[\beta_1 - \beta_0]$.

[4] In the literature, the endowment effect is also known as the *composition effect* while the price effect is also referred to as the *structural effect*. See Fortin, Lemieux and Firpo (2011) for the use of this terminology.

Expressions (3) and (4) provide a simple way of dividing the endowment and the price effects into the contribution of a single covariate or a group of covariates as needed.

The above components are easily computed by replacing the expected values by the corresponding sample means and the coefficients associated with the covariates by their OLS estimates. An estimate of the endowment effect is:

$$\hat{\Delta}_X^{\mu} = (\bar{x}_1 - \bar{x}_0)\hat{\beta}_0 = \sum_{k=1}^{m}(\bar{x}_{1k} - \bar{x}_{0k})\,\hat{\beta}_{0k} \qquad (5)$$

Similarly, for the price effect, we have the following expression.

$$\hat{\Delta}_S^{\mu} = \bar{x}_1\left(\hat{\beta}_1 - \hat{\beta}_0\right) = \left(\hat{\beta}_{11} - \hat{\beta}_{01}\right) + \sum_{k=2}^{m}\bar{x}_{1k}\left(\hat{\beta}_{1k} - \hat{\beta}_{0k}\right) \qquad (6)$$

Interpretation

As Fortin, Lemieux and Firpo (2011) point out, there is a powerful analogy between the Oaxaca-Blinder decomposition method and treatment effect analysis[5]. Treatment impact analysis seeks to identify and estimate the average effect of treatment (i.e. intervention) on the treated (i.e. those exposed to an intervention) on the basis of the difference in average outcomes between the treated and a comparison group. In that context, t indicates treatment status. It is equal to 1 for the treated and 0 for the untreated (the comparison or control group). The expression, $\Delta_O^{\mu} = [E(y_1) - E(y_0)]$, can therefore be interpreted as the difference in average outcomes between the treated and untreated. Under the assumptions[6] underlying the basic Oaxaca-Blinder method, it is clear that this difference is due to differences in observable characteristics (i.e. the composition effect) and in treatment status. The part due to the difference in treatment status is known as the

[5] Indeed, these authors provide a systematic interpretation of decomposition methods within the logic of program impact evaluation.

[6] According to Fortin, Lemieux and Firpo (2011), these assumptions include the following: (1)Mutually exclusive groups;(2) The outcome structure is an additively separable function of characteristics; (3) Zero conditional mean for unobservables given observed characteristics;(4)Common support for the distributions of characteristics across groups (to rule out cases where arguments of the outcome function may differ across groups; (5) Simple counterfactual treatment, meaning that the outcome structure of one group is assumed to be a counterfactual for the other group. This last assumption rules out general equilibrium effects so that observed outcomes for one group or time period can be reasonably used to construct counterfactuals for the other group or time period. The Oaxaca-Blinder method therefore follows a partial equilibrium approach.

average treatment effect on the treated (ATET) and is in fact equal to the structural or price effect.

Note that the conventional approach to impact evaluation also relies on *ceteris paribus* variation of treatment in order to identify its average effect on the treated. Within that logic, the composition effect is equivalent to *selection bias* that must be driven to zero by the use of randomization, propensity score matching or similar methods. Randomization ensures that the distribution of observed and unobserved characteristics is the same for both the treated and the control group. By balancing observed and unobserved characteristics between the groups prior to the administration of treatment, randomization guarantees that the average difference in outcome between the two groups is due to treatment alone, hence the causal interpretation given to this parameter under those circumstances. In other words, the first term on the right hand side of equation (2), that is the endowment effect or selection bias, is equal to zero under random assignment to treatment and full compliance[7]. It is clear that randomization is designed to implement a ceteris paribus variation in treatment.

In the context of observational studies where the investigator does not have control over the assignment of subjects to treatment, the determination of the causal effect of treatment hinges critically on the understanding of the underlying *treatment assignment or selection mechanism* which must explain how people end up in alternative treatment states. The assumption of selection on observables (also known as *ignorability*) is often invoked to implement *ceteris paribus* identification of the average treatment effect through conditioning by stratification. Basically, conditioning by stratification entails comparing only those subjects with the same value of covariates x across the two groups (treated and untreated). This type of selection of individuals from the two groups is known as *matching*.

[7] Heckman and Smith (1995) explain that the mean outcome of the control group provides an acceptable estimate of the counterfactual mean if randomization does not alter the pool of participants or their behavior, and if no close substitutes for the experimental program are readily available. These authors further note that randomization does not eliminate selection bias, but rather balances it between the two samples (participants and nonparticipants) so that it cancels out when computing mean impact. There would be randomization bias if those who participate in an experiment differ from those who would have participated in the absence of randomization. Furthermore, substitution bias would occur if members of the control group can easily obtain elsewhere close substitutes for the treatment.

There is a potential dimension problem associated with matching when there are many observable characteristics taking many values. Insisting on conditioning based on exact values can lead to too few observations in each subgroup characterized by these observables. This dimensionality problem can be resolved by matching on the *propensity score,* that is, the conditional probability of receiving treatment given observable characteristics (Rosenbaum and Rubin 1983).

The analogy between treatment effect analysis and the Oaxaca-Blinder decomposition method has been extremely useful for the development of flexible estimation methods for endowment and structural effects. Fortin, Lemieux and Firpo (2011) explain that selection on observables implies that the conditional distribution of unobservable factors is the same in both groups (treated and comparison). They further note that, while this assumption is weaker than the zero conditional mean assumption[8] used in the standard Oaxaca-Blinder decomposition, it is enough to secure identification and consistent estimation of the ATET and hence the structural effect, Δ_S^μ, (in the Oaxaca-Blinder framework). These authors give the example of education and unobservable ability. They explain that if education and ability are correlated, this creates an endogeneity problem that prevents a linear regression of earnings on education to produce consistent estimates of the structural parameters measuring the return to education. Yet the aggregate decomposition remains valid as long as the correlation between ability and education is the same in both groups.

A major implication of the difference in identification assumptions between the traditional Oaxaca-Blinder approach and treatment effect analysis is that consistent estimators of the ATET such as inverse probability weighing (IPW) and matching can be used to estimate the structural effect (Δ_S^μ) even if the underlying relationship between the outcome and covariates is not linear. Given such an estimate, the composition effect can be calculated as a residual from the overall mean difference as follows: $\Delta_X^\mu = \Delta_O^\mu - \Delta_S^\mu$. In

[8] Recall that, the identification of the two components of the aggregate Oaxaca-Blinder decomposition relies on the zero conditional mean assumption for the unobservable factors stated as: $E(\epsilon|x) = 0$. This condition is what allows the analyst to claim that on average, variation in x is unrelated to variation in the unobservables, a manifestation of *ceteris paribus variation*.

particular, decomposition methods based on this weighting procedure are known to be efficient.

Limitations

While treatment effect analysis can help with the identification and estimation of the structural effect, it is important to note that there are two basic reasons why this effect does not necessarily inherit the causal interpretation generally enjoyed by the ATET. The first reason stems from the fact that in many cases, group membership is not the result of a choice or an exogenous assignment but a consequence of an intrinsic characteristic such as gender or race. The other important reason is that many of the observable covariates are not equivalent to the so-called pre-treatment variables that are not supposed to be affected by the treatment (Fortin, Lemieux and Firpo 2011).

Fortin, Lemieux and Firpo (2011) point out two other important limitations of the standard Oaxaca-Blinder decomposition method. The contribution of each covariate to the structural effect is highly sensitive to the choice of the omitted group when the explanatory variables include a categorical variable. Jann (2008) discusses possible solutions to this problem. The second limitation stems from the fact that the decomposition provides consistent estimates only under the assumption that the conditional expectation is linear. Under the linearity assumption, the counterfactual average when t=1 is simply equal to: $E(x_1|t = 1) \cdot \beta_0$. This is estimated by the cross-product of sample means of characteristics for t=1 with the relevant OLS coefficients from t=0. The corresponding estimate is: $\overline{x}_1\hat{\beta}_0$. The counterfactual mean outcome will not be equal to this term when linearity does not hold. One possible solution is to reweight the sample for t=0 using the inverse probability method[9] and to compute the counterfactual mean outcome on the basis of statistics from the reweighted counterfactual sample. Let $\bar{x}_0^c$ be the vector of the means of adjusted covariates in t=0, and $\hat{\beta}_0^c$ the corresponding least squares coefficients. Then the correct counterfactual mean outcome when the linearity assumption does not hold is: $\bar{x}_0^c\hat{\beta}_0^c$. This is the term to add to and subtract from the empirical version of the overall difference

[9] We will come back to this point in the next subsection.

in mean outcome to get the appropriate estimates of the endowment and structural effects when the linearity assumption fails.

2.3. A Generalization of the Oaxaca-Blinder Decomposition

As noted above, the standard Oaxaca-Blinder decomposition method focuses on differences in mean outcomes between two groups and relies on some stringent assumptions to identify the endowment and structural effects. We now consider how to extend the logic underlying this method to the decomposition of differences in distributional statistics other than the mean such as poverty or inequality measures. We consider both aggregate and detailed decompositions.

Aggregate Decomposition

Just as in the case of the basic Oaxaca-Blinder method, we are interested in decomposing a change in some distributional statistic, say θ, from the base period t=0 to the end period t=1. As noted in the introduction, all distributional statistics such as the mean, quantiles, the variance, poverty and inequality measures, can be viewed as functionals of the underlying outcome distribution. Thus, the principle of decomposition presented here applies to all of them. Let $F_{y_0|t=0}$ stand for the outcome distribution observed in the initial period and $F_{y_1|t=1}$ that observed in the final period. The overall difference in the distribution of outcomes between states 0 and 1 can be written in terms of θ(F) as follows (Fortin, Lemieux and Firpo 2011).

$$\Delta_O^{\theta} = \theta\left(F_{y_1|t=1}\right) - \theta\left(F_{y_0|t=0}\right) \qquad (7)$$

Equation (7) characterizes the domain of the decomposition methods in this paper. As far as the scope is concerned, most micro methods seek to decompose this overall difference on the basis of the relationship between the outcome variable and individual or household characteristics. The following equation represents a general expression of that relationship.

$$y_t = \varphi_t(x_t, \varepsilon_t),\ \ t = 0, 1. \qquad (8)$$

Equation (8) suggests that conditional on the observable characteristics, x, the outcome distribution depends only on the function $\varphi_t(\cdot)$ and the distribution of the unobservable characteristics ε. Thus there are four potential terms in the scope of micro-decomposition methods based on this framework. Differences in outcome distributions between the two periods may be due to: (i) differences in the returns to observable characteristics given the functions defining the outcome structure, (ii) differences in the returns to unobservable characteristics also defined by the structural functions (iii) differences the distribution of observable characteristics, and (iii) differences in the distribution of unobservable characteristics.

Given the potential scope implied by the outcome model (8), the next step is to impose enough restrictions in order to identify the factors of interest. In general these restrictions are imposed on the form of the outcome functions, $\varphi_t(\cdot)$, and on the joint distribution of the observable and unobservable characteristics, x and ε. Let's maintain the assumptions of mutually exclusive groups, simple counterfactual treatment and common support that also underlie the Oaxaca-Blinder decomposition. Fortin, Lemieux and Firpo (2011) explain that, under the general outcome model presented in equation (8), it is impossible to distinguish the contribution of the returns to observables from that of unobservables. These two terms can therefore be lumped in a single term, the structural effect noted Δ_S^θ. Let Δ_X^θ stand for the endowment effect and $\Delta_\varepsilon^\theta$ for the effect associated with differences in the distribution of unobservables. The issue now is to identify these three effects so that they account for the overall difference described by equation (7).

Let $y_{0|t=1}$ be the outcomes that would have prevailed in period 1 if individual characteristics in that period had been rewarded according to $\varphi_0(\cdot)$. Let $F_{y_0|t=1}$ stand for the corresponding distribution and $\theta\left(F_{y_0|t=1}\right)$ the corresponding value of the statistic of interest. Assuming ignorability in addition to the previously maintained assumptions, the endowment effect is identified by: $\Delta_X^\theta = \left[\theta\left(F_{y_0|t=1}\right) - \theta\left(F_{y_0|t=0}\right)\right]$. The validity of this identification rests on that of the assumption of ignorability which implies that the conditional distribution of unobservable factors is the same in both states of the world. Hence $\Delta_\varepsilon^\theta = 0$. Under the same set of assumptions, the structural effect is due solely to differences in the functions defining the outcomes[10]. This effect is identified by the following expression: $\Delta_S^\theta = \left[\theta\left(F_{y_1|t=1}\right) - \theta\left(F_{y_0|t=1}\right)\right]$.

Given the outcome model represented by equation (8), assuming mutually exclusive groups, common support, simple counterfactual treatment and ignorability, we can decompose the distributional difference in equation (7) by adding to and subtracting from it the counterfactual outcome $\theta\left(F_{y_0|t=1}\right)$. This leads to the following expression.

$$\Delta_O^\theta = \left[\theta\left(F_{y_0|t=1}\right) - \theta\left(F_{y_0|t=0}\right)\right] + \left[\theta\left(F_{y_1|t=1}\right) - \theta\left(F_{y_0|t=1}\right)\right] \tag{9}$$

where the first term on the right hand side is the endowment effect and the second is the structural effect (Fortin, Lemieux and Firpo 2011). In the context of poverty analysis, if P stands for the poverty measure of interest, then equation (9) implies that observed changes in poverty can be decomposed as follows.

$$\Delta_O^P = \left[P\left(F_{y_0|t=1}\right) - P\left(F_{y_0|t=0}\right)\right] + \left[P\left(F_{y_1|t=1}\right) - P\left(F_{y_0|t=1}\right)\right] \tag{10}$$

DiNardo, Fortin and Lemieux (1996) show that the counterfactual distribution, $F_{y_0|t=1}$, can be estimated by properly reweighing the distribution of covariates in period 0. One can express the resulting counterfactual distribution as follows[11].

$$F_{y_0|t=1}(y) = \int F_{y_0|x_0}(y|x)\mathrm{w}(x)dF_{x_0}(x) \tag{11}$$

where the reweighing factor is equal to: $\mathrm{w}(x) = \frac{dF_{x_1}(x)}{dF_{x_0}(x)} = \frac{P(t=1|x)}{1-P(t=1|x)} \cdot \frac{1-\pi}{\pi}$. These weights are proportional to the conditional odds of being observed in state 1. The proportionality factor depends on π which is the proportion of cases observed in state 1. One can easily

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[10] To see this, note that $y_{1|t=1} = \varphi_1(x_1, \varepsilon_1)$ and $y_{0|t=1} = \varphi_0(x_1, \varepsilon_1)$.

[11] To further appreciate the importance of the identifying assumptions, note that the process of reweighing adjusts the distribution of the covariates x in period t=0 so that it becomes similar to that in period t=1. For this adjustment to help us identify the terms of the decomposition it must be a *ceteris paribus* adjustment. Since $y_0 = \varphi_0(x, \varepsilon)$, the ceteris paribus condition would be violated if changing the distribution of x also changed either the function $\varphi_0(\cdot)$ or the conditional distribution of ε given x. This would confound the impact of the adjustment and the decomposition would be meaningless. Changes in the structural function are ruled out by the simple treatment assumption (no general equilibrium effects) while those in the conditional distribution of ε are ruled out by the ignorability assumption. Under this circumstances, we expect the conditional distribution of y_0 given x to be invariant with respect to adjustments in the distribution of the observable factors x. See Fortin, Lemieux and Firpo (2011) for a more formal presentation of this argument.

compute the reweighing factor on the basis of a probability model such as logit or probit. Furthermore, if one is interested only in the aggregate decomposition of the variation in a distributional statistic, then all that is needed are an estimate of the relevant counterfactual distribution and the corresponding value of the statistic in question.

The decomposition presented in equation (9) is based on a nonparametric identification and can be estimated by the Inverse Probability Weighing (IPW) method implied by equation (11). Nonparametric methods allow analysts to decompose changes in distributional statistics into endowment and structural effects without having to assume a functional form for the outcome model. The downside is that one cannot separate the respective contributions of the observable and unobservable factors into the structural effect, nor can one account for changes in agents' behavior. In the next section we consider a way of separating the contribution of unobservables from that of observables, and in section 4 we review methods that have been proposed to account for behavioral responses.

Detailed Decomposition

Fortin, Lemieux and Firpo (2011) explain that a decomposition approach provides a detailed decomposition when it allows one to apportion the composition effect or the structural effect into components attributable to each explanatory variable. The contribution of each explanatory variable to the composition effect is analogous to what Rothe (2010) calls a "*partial composition effect*"[12]. As discussed earlier, this is easily accomplished in the context of the classic Oaxaca-Blinder decomposition because of the two underlying assumptions of linearity and zero conditional mean for the unobservable factors. Recentered influence function (RIF) regression that we review next also offers a possibility to perform detailed decomposition in a way that mimics the basic Oaxaca-Blinder approach.

RIF regression offers a simple way of establishing a direct link between a distributional statistic and individual (or household) characteristics. This link offers an opportunity to perform

[12] This is the effect of a counterfactual change in the marginal distribution of a single covariate on the unconditional distribution of an outcome variable, *ceteris paribus*. Rothe (2010) interprets the *ceteris paribus* condition in terms of rank invariance. In other words, the counterfactual change in the marginal distribution of the relevant covariate is constructed in such a way that the joint distribution of ranks is unaffected.

both aggregate and detailed decompositions for any such statistic for which one can compute an *influence function* (Fortin, Lemieux and Firpo 2011). In the literature, the derivative of a functional θ(F) is called the influence function of θ at F. The function measures the relative effect of a small perturbation in F on θ(F). In that sense, it is a measure of robustness[13]. Firpo, Fortin and Lemieux (2009) define the recentered or rescaled influence function (RIF) as the leading terms of a von Mises (1947) linear approximation of the associated functional[14]. It is equal to the functional plus the corresponding influence function.

It is known that the expected value of the influence function is equal to zero. This implies that the expected value of the RIF is equal to the corresponding distributional statistic. In other words, $\theta\left(F_y\right) = E[RIF(y;\ \theta)]$. By the law of iterated expectations the distributional statistic of interest can be written as the conditional expectation of the rescaled influence function (given the observable covariates, x). This is the RIF regression that, for $\theta\left(F_y\right)$, can be expressed as: $E[RIF(y;\theta)|x]$. The distributional statistic $\theta\left(F_y\right)$ can therefore be expressed in terms of this conditional expectation as follows (Firpo, Fortin, Lemieux 2009).

$$\theta(F_y) = \int E[RIF(y;\ \theta)|x]dF(x) \tag{12}$$

This expression suggests that to assess the impact of covariates on $\theta\left(F_y\right)$, one needs to integrate over the conditional expectation $E[RIF(y;\theta)|x]$. This can be easily done using regression methods. In particular, one can model this conditional expectation as a linear function of observable covariates as : $E[RIF(y;\theta)|x] = x\beta$, and apply OLS to the following equation.

$$RIF(y;\theta) = x\beta + \varepsilon \tag{13}$$

Fortin, Lemieux and Firpo (2011) explain that the expected value of the linear approximation of the RIF regression is equal to the expected value of the true conditional expectation because the expected value of the approximation error is zero. This fact makes the extension of the standard Oaxaca-Blinder decomposition to RIF regressions both simple and meaningful.

[13] Wilcox (2005) explains that continuity alone confers only qualitative robustness to the statistic under consideration. A continuous function is relatively unaffected by small shifts in its argument. Similarly, differentiability is related to infinitesimal robustness in the sense that, if a function is differentiable and its derivative is bounded, then small variations in the argument will not result in large changes in the function. Thus a search for robust statistics can focus on functionals with bounded derivatives.

[14] This is analogous to the approximation of a differentiable function at a point by a Taylor's polynomial.

Applying the standard Oaxaca-Blinder approach to equation (13) we find that the endowment effect can be written as follows.

$$\Delta_X^{\theta} = [E(x|t=1) - E(x|t=0)] \cdot \beta_0 \tag{14}$$

The corresponding structural effect is

$$\Delta_S^{\theta} = E(x|t=1) \cdot (\beta_1 - \beta_0) \tag{15}$$

This decomposition may involve a bias since the linear specification is only a local approximation that may not hold in the case of large changes in covariates[15]. The solution to this problem, consistent with our discussion in subsection 2.2, is to combine reweighing with RIF regression (see Fortin, Lemieux and Firpo 2011) for details.

3. Endowment and Price Effects along the Entire Outcome Distribution

The presentation of the basic framework in section 2 focuses on the decomposition of aggregate statistics. As noted in the introduction, these statistics provide little information about the heterogeneity of impacts underlying aggregate outcomes. This section therefore applies the same framework to the identification and estimation of the aggregate endowment and price effects along the whole distribution of outcomes. All the methods reviewed in this section are purely statistical in the sense that they all rely on models of the conditional distribution of outcomes given the covariates. We consider in turn the decomposition of differences in density functions and across quantiles. The decomposition across quantiles also allows the analyst to express changes in poverty in terms of endowment and price effects. The decomposition of changes in density functions relies on nonparametric methods. In the case of quantiles, we focus on parametric methods. Along the way, we note circumstances under which the contribution of unobservables in the structural effect can be distinguished from that of observables.

3.1 Differences in Density Functions

For decomposition purposes, one needs a model that links the outcome of interest to household characteristics. To focus on differences in density functions, we maintain that the outcome variable y has a joint distribution with characteristics, x. This distribution is

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[15] In particular, $\beta_1$and β_0 may differ just because their estimation is based on different distributions of the covariates x, even if the outcome structure remains unchanged (Firpo, Fortin and Lemieux 2009).

characterized by the following joint density function: $J_t(y,x), t = 0,1$. The generalization of the Oaxaca-Blinder decomposition considered here requires the marginal distribution of y noted as: $f_t(y)$. This marginal density function can be obtained by integrating the covariates x out of the joint density. Furthermore, the factorization principle allows one to write the joint density as a product of the distribution of y conditional on x, $g_t(y|x)$, and the joint distribution of characteristics, $h_t(x)$. These are the two factors underpinning the decomposition. Any change in the marginal outcome distribution induced by a variation in the distribution of observed characteristics (*ceteris paribus*) represents the *endowment effect*, while any change in the distribution associated with a (*ceteris paribus*) variation in the conditional distribution is interpreted as the *price-behavioral effect* (Bourguignon and Ferreira 2005).

To see clearly what is involved[16], we express the joint density function as a product of the two underlying functions: $J_t(y,x) = g_t(y|x)h_t(x), t = 0,1$. On the basis of this factorization, we can write the marginal density of y in a way that facilitates the expression and interpretation of the decomposition results, that is: $f_t(y) \equiv f_{gt}^{ht}(y)$. Thus the observed change in the outcome distribution between the two periods can be stated as follows.

$$\Delta f = f_1(y) - f_0(y) \equiv f_{g1}^{h1}(y) - f_{g0}^{h0}(y) \tag{16}$$

We can add to and subtract from the difference defined in (16) the following counterfactual[17]: $f_{g0}^{h1}(y)$. This is the marginal density function that would obtain if the conditional distribution were that of period 0, and the joint distribution of characteristics

[16] This account draws on Essama-Nssah and Bassolé (2010)

[17] To clarify our notation, we consider the simplest case where x represents a single characteristic. No loss of generality is involved. The marginal distribution of y is equal to $f_t(y) = \int_0^{mx} J_t(y,x)dx$, where mx stands for the maximum value of x. Equivalently, $f_t(y) = f_{gt}^{ht}(y) = \int_0^{mx} g_t(y|x)h_t(x)dx$. The counterfactual used in equation (17) is therefore defined as follows: $f_{g0}^{h1} = \int_0^{mx} g_0(y|x)h_1(x)dx$. This expression can be derived from the marginal outcome distribution in the initial period, $f_0(y) = \int_0^{mx} g_0(y|x)h_0(x)dx$, by replacing $h_0(x)$ with $h_1(x)$. As explained in footnote 11, for this operation to lead to a meaningful counterfactual, two invariance conditions must be met. The conditional distributions $g_t(y|x)$ must be invariant with respect to changes in the marginal distribution of observables, $h_t(x)$. This would be the case if there are no general equilibrium effects. The distribution of unobservables must be at least conditionally independent of that of observables. Ignorability guarantees this.

that prevailing in period 1. This transformation leads us to the following generalized decomposition of changes in the marginal density of y.

$$\Delta f = \left[f_{g0}^{h1}(y) - f_{g0}^{h0}(y)\right] + \left[f_{g1}^{h1}(y) - f_{g0}^{h1}(y)\right] \tag{17}$$

The configuration of the indices (subscripts and superscripts) for the marginal distributions involved in (17) suggests an interpretation of the various components of the decomposition. The first component on the right hand side is the *endowment effect* (based on changes in the joint distribution of observed characteristics). The second component measures the *price-behavioral effect* (linked to the change in the conditional distribution of y which, in fact, also includes the effect of unobservables).

In their study of the role of institutional factors in accounting for changes in the distribution of wages in the U.S., DiNardo, Fortin and Lemieux (1996) demonstrate how to implement empirically the above decomposition using kernel density methods to estimate the relevant density functions. The *histogram* is the oldest and most common density estimator (Silverman 1986), and kernel methods may be viewed as ways of smoothing a histogram. *The basic idea is to estimate the density f(y) by the proportion of the sample that is near y*. One way of proceeding is to choose some interval or "band" and to count the points in the band around each y and normalize the count by the sample size multiplied by the bandwidth. The whole procedure can be viewed as sliding the band (or window) along the range of y, calculating the fraction of the sample per unit within the band, and plotting the result as an estimate of the density at the mid-point of the band (Deaton 1997)[18].

The kernel estimate of the density function $f_t(y)$ can be written as follows.

$$\widehat{f}_t(y) = \frac{1}{hn_t}\sum_{i=1}^{n_t} K\left(\frac{y_{it}-y}{h}\right), t = 0, 1. \tag{18}$$

[18] Deaton (1997) further explains that the size of the bandwidth is inversely related to the sample size. The larger the sample size, the smaller the bandwidth. To obtain a consistent estimate of the density at each point, the bandwidth must become smaller at a rate that is slower than the rate at which the sample size is increasing. However, with only a few points, we need large bands to be able to get any points in each. By widening the bands, we run the risk of biasing the estimate by bringing into the count data that belong to other parts of the distribution. Hence, the increase in the sample size does two things. It allows the analyst to reduce the bandwidth and hence the bias in estimation (due to increased mass at the point of interest), it also ensures that the variance will shrink as the number of points within each band increases.

where h is the bandwidth representing the smoothing parameter, n_t is the sample size for period t, y is the focal point where the density is estimated, and K(·) is the kernel function. A kernel function is essentially a weighting function chosen in such a way that more weight is given to points near y and less to those far away. In particular, it will assign a weight of zero to points just outside and just inside the band. As a weighting function, the kernel function should satisfy four basic properties: (i) positive, (ii) integrate to unity over the band, (iii) symmetric around zero so that points below y get the same weight as those an equal distance above, (iv) decreasing in the absolute value of its argument. The most common kernel functions used in empirical work are the Gaussian and the Epanechnikov kernel[19].

The counterfactual density function that is the *linchpin* of the decomposition presented in equation (17) can be written in a manner analogous to the distribution functions underlying the decomposition presented in equation in (9)[20]. In other words, $f_{g0}^{h1}(y) = f_{y_0|t=1}$. This density can be estimated by reweighing the kernel estimate for period 0 using the same weighing function as the one underlying the counterfactual distribution defined in equation (11). The resulting expression is:

$$\hat{f}_{y_0|t=1}(y) = \frac{1}{hn_0}\sum_{i=1}^{n_0} w(x)K\left(\frac{y_{i0}-y}{h}\right) \tag{19}$$

Machado and Mata (2005) propose a semi-parametric approach to estimating the density functions needed in the above decomposition. Their approach is based on a two-step procedure that allows them to derive marginal density functions from the conditional quantile process that fully characterizes the conditional distribution of y given the covariates x. Specifically, these authors model the conditional distribution of y given x by a linear conditional quantile function as follows.

$$q_\tau(y_t|x_t) = x_t\beta_t(\tau), \tau \in (0,1), t = 0,1 \tag{20}$$

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[19] Deaton (1997) argues that the choice of the bandwidth or the smoothing parameter is more important than that of the kernel function. Essentially, estimating densities by kernel methods is an exercise in smoothing the sample observations into an estimated density. The bandwidth controls the amount of smoothing achieved. Over-smoothed estimates are biased, while under-smoothed ones are too variable.

[20] The equivalent expression for the decomposition is: $\Delta f = \left[f_{y_0|t=1} - f_{y_0|t=0}\right] + \left[f_{y_1|t=1} - f_{y_0|t=1}\right]$.

The second step in the approach entails estimating the marginal density function of y that is consistent with the conditional quantile process defined by (20). This is achieved by running the following algorithm: (i) Draw a random sample of size m from a uniform distribution on [0, 1] to get τ_j for j=1, 2, , m; (ii) For each τ_j, use available data to estimate the quantile regression model and get m estimates of coefficients $\hat{\beta}_t(\tau_j), j = 1, 2, \ldots, m$; (iii) Given that x_t is a (n_t x k) matrix of data on covariates, draw a random sample of size m from the rows of x_t and denote each such sample by x_{jt}^s; (iv) The corresponding values of the outcome variable are given by: $y_{jt}^s \equiv x_{jt}^s \hat{\beta}_t(\tau_j), j = 1, 2, \ldots, m$. The validity of this procedure stems from the *probability integral transformation theorem* which states that, if u is a random variable uniformly distributed over [0, 1], then $y = F^{-1}(u)$ is distributed like F. Here τ_j is assumed to be a realization of $F_{y_t|x_t}$. Given model (20), the corresponding conditional quantile regression model can be written as (Fortin, Lemieux and Firpo 2011):

$$q_{\tau_j}(y_t|x_t) = F_{y_t|x_t}^{-1}(\tau_j, x_t) = x_t \beta_t(\tau_j), t = 0, 1 \tag{21}$$

A modified version of the above algorithm leads to the critical counterfactual upon which the decomposition is based. Recall that the counterfactual of interest is the density function of the outcome in period 1 assuming that the characteristics of that period had been rewarded according to the system prevailing in period 0. This counterfactual can be estimated by applying the above algorithm to the data for period 0, except that at stage (iii) covariates must be drawn from data for period 1. On the basis of equation (21), the conditional regression model associated with this counterfactual is the following.

$$q_{\tau_j}(y_0|x_1) = F_{y_0|x_0}^{-1}(\tau_j, x_1) = x_1 \beta_0(\tau_j) \tag{22}$$

As noted by Fortin, Lemieux and Firpo (2011), this approach is computationally demanding. They suggest a simplification based on the estimation of a large number of quantile regressions (say 99) instead of using the random process. The conditional quantile function can then be inverted to obtain the conditional cumulative distribution which must be averaged over the empirical distribution of the covariate to yield unconditional distribution function. In fact, Machado and Mata (2005) acknowledge that this is a viable alternative to their method.

3.2 Differences across Quantiles

One can also work with quantiles instead of density functions (or equivalently, distribution functions) to decompose changes along the entire outcome distribution. Since the decomposition must be based on marginal distributions, one needs to work with marginal quantiles, not conditional ones. There is a variety of ways to go about it. Recall that the general decomposition presented in section 2 about a distributional statistic $\theta(F_y)$ applies to any statistic including quantiles. In that case, the counterfactual distribution is derived from equation (11). Alternatively, marginal quantiles can be derived from equations (20) and (21) based on the Machado and Mata (2005) procedure or by numerical integration as proposed by (Melly 2005).

To link conditional quantiles to marginal quantiles, Angrist and Pischke (2009) start from the observation that the proportion of the population below q_τ conditional on x is equal to the proportion of conditional quantiles that are below q_τ. Let $I(\cdot)$ be the indicator function that takes a value of 1 if its argument is true and 0 otherwise. Again, let $F_{y|x}()$ stand for the conditional cumulative distribution function (CDF) of y given x. Thus the proportion of the population for which the outcome y is less than q_τ is equal to: $F_{y|x}(q_\tau|x) = \int_0^1 I\left[F_{y|x}^{-1}(\tau|x) < q_\tau\right]d\tau$, where the term on the right hand side is equal to the proportion of conditional quantiles that are below q_τ. On the basis of equation (20), we can rewrite this proportion as : $F_{y|x}(q_\tau|x) = \int_0^1 I[x\beta(\tau) < q_\tau]d\tau$. The marginal distribution of y, $F_y()$ from which one derives marginal quantiles, is obtained by integrating the conditional distribution over the whole range of the distribution of the covariates (Melly 2005). The resulting expression is: $F_y(q_\tau) = \int \left(\int_0^1 I[x\beta(\tau) < q_\tau]d\tau\right) dF_x$. The sample analog of this expression based on an estimation of quantile regressions at every percentile for a sample of size n is given by the following expression (Angrist and Pischke 2009).

$$\hat{F}_y(q_\tau) = \frac{1}{n}\sum_{i=1}^{n}\left(\frac{1}{100}\sum_{\tau=0}^{1} I(x_i\beta_\tau < q_\tau)\right) \tag{23}$$

The marginal quantile corresponding to the above estimator of the marginal distribution of the response variable is obtained by inverting (23). We note these marginal quantiles as: $q_\tau\big(x_t, \hat{\beta}_t(\tau)\big) = inf\{q: \hat{F}_y(q_\tau)\}$.

The generalized Oaxaca-Blinder decomposition described by equation (9) can equivalently be stated in terms of these marginal quantiles. The observed change in the marginal distribution of the response variable is now written as: $\Delta q_\tau = q_\tau\big(x_1, \hat{\beta}_1(\tau)\big) - q_\tau\big(x_0, \hat{\beta}_0(\tau)\big)$. To distinguish the endowment effect from the price effect, we subtract from and add to this expression the following counterfactual outcome: $q_\tau\big(x_1, \hat{\beta}_0(\tau)\big)$. This counterfactual involves the characteristics of period 1 evaluated with the prices (coefficients) of period 0. The corresponding decomposition analogous to expression (9) is the following.

$$\Delta q_\tau = \Big[q_\tau\big(x_1, \hat{\beta}_0(\tau)\big) - q_\tau\big(x_0, \hat{\beta}_0(\tau)\big)\Big] + \Big[q_\tau\big(x_1, \hat{\beta}_1(\tau)\big) - q_\tau\big(x_1, \hat{\beta}_0(\tau)\big)\Big] \qquad (24)$$

Consistent with equation (9), the first term on the right hand side of (24) is the endowment effect at the τ^{th} quantile while the second term measures the price effect at the same location.

As noted in section 2, one can use RIF regression to perform detailed decomposition of differences across quantiles. Firpo, Fortin and Lemieux (2009) show that the rescaled influence function of the τ^{th} quantile of the distribution of y is the following[21]:

$$RIF(y;\ q_\tau) = q_\tau + IF(y;\ q_\tau) = q_\tau + \frac{[\tau - I(y \leq q_\tau)]}{f_y(q_\tau)} \qquad (25)$$

Where I(·) is an indicator function for whether the outcome variable y is less than or equal to the τ^{th} quantile and $f_y(q_\tau)$ is the density function of y evaluated at the τ^{th} quantile. Essama-Nssah et

[21] Essama-Nssah and Lambert (2011) show how to derive the influence function of a functional from the associated directional derivative. They present a collection of influence functions for social evaluation functions commonly used in assessing the distributional and poverty impact of public policy. Their catalog includes, among others, influence functions and recentered influence functions for the mean, the τ^{th} quantile, the Gini coefficient, the Atkinson index of inequality, the class of additively separable poverty measures defined in equation (27) below, the growth incidence curve ordinate, the Lorenz curve and generalized Lorenz curve ordinates, the TIP curve ordinate and some measures of pro-poorness associated with the Foster, Greer and Thorbecke (1984) family of poverty measures.

al. (2010) apply this methodology to account for heterogeneity in the incidence of economic growth in Cameroon.

At this stage we pause, to consider the implications of this decomposition for poverty comparison over time. One can use equation (24) repeatedly to decompose, the first 99 quantiles (percentiles) of the outcome distribution of interest. This means that we can decompose the growth incidence curve[22] (GIC) in a component due to the endowment effect and another due to the price effect. Formally, we express this decomposition as follows.

$$g(y) = g_X(y) + g_S(y) \tag{26}$$

where the first component on the right hand side of (26) is the endowment effect for the GIC and the second term is the corresponding structural effect.

For the class of additively separable poverty measures, a change in poverty over time can be written as a weighted sum of points on the growth incidence curve (Essama-Nssah and Lambert 2009, Ferreira 2010). Therefore, change in poverty over time inherits the decomposability of the growth incidence curve. To see what is involved here, note that the class of additively separable poverty measures is defined by the following expression:

$$P(F;z) = \int_0^z \psi(y|z)dF(y) \tag{27}$$

where F stands for the distribution of a continuous outcome variable y and z is the poverty line. This expression makes it clear that the poverty measure P(·) can be viewed as a functional of F. In other words, a poverty measure reveals the level of aggregate poverty associated with a distribution and a poverty line. The term $\psi(y|z)$ is a convex and decreasing function measuring deprivation for an individual with a level of economic welfare equal to y. This function is equal to zero when the welfare indicator is greater or equal to the poverty line. For members of the additively separable class defined by (27), a change in poverty associated with the growth pattern depicted by the incidence curve g(y) is given by the following expression:

[22] Ravallion and Chen (2003) define the growth incidence curve as the growth rate of an indicator of welfare (income or consumption) y at the p[th] percentile point of its distribution. The outcome at that point can be noted a y(p).

$$dP = \int_0^z y\psi'(y|z)g(y)dF(y) \tag{28}$$

where ψ' is the first-order derivative of the indicator of individual deprivation. On the basis of equation (26), the variation in poverty defined by equation (28) can be equivalently expressed as follows.

$$dP = \int_0^z y\psi'(y|z)g_X(y)dF(y) + \int_0^z y\psi'(y|z)g_S(y)dF(y) \tag{29}$$

The first term on the right hand side of (29) represents the endowment effect based on the endowment effect for the GIC. Similarly, the second term is associated with the price or structural effect for the GIC. In terms of equation (24), note that $\Delta q_\tau = yg(y) = yg_x + yg_S$. Again, in this case, the corresponding detailed decomposition of the GIC carries over to variations in poverty outcomes that are based on additively separable poverty measures. This is in fact true for all additively separable social evaluation functions (e.g. Atkinson welfare function).

3.3 Accounting for the Contribution of Unobservables

Recall that, on the basis of equation (8), there are at least two potential components to the contribution of unobservable characteristics into changes in the outcome distribution. The first relates to changes in the returns to unobservables and the second to the distribution of these characteristics. All the decomposition methods discussed so far lump the first component together with the returns to observables in the structural effect. Furthermore, the contribution of changes in the distribution of unobservable characteristics is ruled out either by the ignorability assumption or by the zero conditional mean assumption. The issue now is: Under what conditions can we identify these effects that, up to now, have been swept under the rug so to speak?

Juhn, Murphy and Pierce (1993) assume *additive linearity* for the outcome model and *conditional rank preservation* in order to decompose differences in outcome distributions in a way that accounts for the contribution of unobservables[23]. Under

[23] In the context of treatment effect analysis, the assumption of *rank preservation*, also known as *rank invariance*, is used to identify quantile treatment effects (QTE). The assumption implies that, given two mutually exclusive states of the world, the outcome at the τ^{th} quantile of the outcome distribution in one state

additive linearity, the function defining the outcome variable is separable in x and ε. We can therefore write the outcome model as follows:

$$y_t = x_t\beta_t + v_t,\ \ t = 0,1 \tag{30}$$

Where $v_t = \zeta_t(\varepsilon)$, some function of unobservable characteristics.

The assumption of conditional rank preservation means that a given individual has the same rank in the distribution of v_0 as in the distribution of v_1, conditional on her observable characteristics. To see this formally, let $F_{v|x}(v_t|x_t)$ stand for the distribution of v_t conditional on x_t. Also, let $\tau_{i0}(x_i) = F_{v|x}(v_{i0}|x_i)$ be the rank of individual i with observed characteristics x_i in the conditional distribution of v_0 given x, and $\tau_{i1}(x_i) = F_{v|x}(v_{i1}|x_i)$ her rank in the conditional distribution of v_1 given x. Conditional rank preservation says that $\tau_{i0}(x_i) = \tau_{i1}(x_i)$. Fortin, Lemieux and Firpo (2011) explain that one can secure conditional rank invariance by assuming ignorability and that the functions v_t are strictly increasing in ε. In other words, these functions are *monotonic*[24].

As expected, separability allows the analyst to construct counterfactuals separately for observables and unobservables. To see what is involved, consider the case of a particular individual, i, with outcome $y_{i1} = x_{i1}\beta_1 + v_{i1}$ in period 1. Let v_{i0}^c represent what the residual part of the outcome would have been, had the unobservable characteristics of this individual been treated as in the initial period, *ceteris paribus*. The corresponding counterfactual for the full outcome is: $y_{i0}^c = x_{i1}\beta_1 + v_{i0}^c$. Comparing this counterfactual with the observed outcome reveals the contribution of changes in the returns to

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has its counterpart at the same quantile of the outcome distribution in the alternative state. Bitler et al. (2006) explain that when this assumption fails, the QTE approach identifies and estimates the difference between the quantiles and not the quantiles of the difference in outcome distributions. Rank preservation is akin to *anonymity* or *symmetry* used to base growth incidence analysis on cross-section data instead of panel data. Anonymity implies that when comparing two outcome distributions, the identity of the individual experiencing a particular outcome is irrelevant (Carneiro, Hansen and Heckman 2002). Thus, a permutation of outcomes between any two individuals in any of the two distributions being compared has no effect on the comparison. One might as well then compare such distributions across quantiles.

[24] Recall that ignorability means that the conditional distribution of ε is the same across groups (or periods). Thus individuals with the same set of observable characteristics find themselves at the same rank in both (conditional) distributions. It is well known that a monotonic transformation preserves order. In fact, Rapoport (1999) defines a monotone transformation as "a formula that changes the numbers of one set to the numbers of another set while preserving their relative positions on the axis of real numbers." Since v_t is obtained from ε through a monotonic transformation $\zeta_t(\cdot)$, rank preservation must therefore follow.

unobservable characteristics of individual i in the overall change in her outcome. We denote this by: $\Delta^y_{S,\sigma} = (y_{i1} - y^c_{i0}) = (v_{i1} - v^c_{i0})$. Next, we replace β_1 with β_0 in the expression for y^c_{i0}. This operation yields the following counterfactual: $y^b_{i0} = x_{i1}\beta_0 + v^c_{i0}$. Let $\Delta^y_{S,\beta} = \left(y^c_{i0} - y^b_{i0}\right)$. This term is equivalent to $\Delta^y_{S,\beta} = x_{i1}(\beta_1 - \beta_0)$ and clearly shows the contribution of changes in the returns to observable characteristics. Thus, separability along with ignorability and monotonicity make it possible to split the structural effect into a component due to changes in the returns to observables and the other linked solely to changes in returns to unobservables. In other words, the total structural effect[25] is equal to: $\Delta^y_S = \Delta^y_{S,\beta} + \Delta^y_{S,\sigma}$. The observable composition effect Δ^y_X can be identified residually from the following expression: $\Delta^y_O - \Delta^y_S = \Delta^y_X + \Delta^y_\varepsilon$ where $\Delta^y_O = (y_{i1} - y_{i0})$. The assumption of conditional independence implies however that $\Delta^y_\varepsilon = 0$. Recall that this assumption implies that the conditional distribution of unobservables does not vary across groups (periods). Therefore, under the prevailing identifying assumptions, the difference between the overall outcome difference and the structural effect identifies the observable composition effect.

The question now is, how does one identify v^c_{i0}? This is where rank preservation comes in. This assumption leads to the following *imputation rule*.

$$v^c_{i0} = F^{-1}_{v_0|x}(\tau_{i1}(x_i)) \tag{31}$$

This imputation rule says that, for individual i in the end period, the counterfactual for the residual outcome is equal to the residual outcome associated with the individual located at the same rank in the conditional distribution of residual outcomes in the base period. In practice, one would estimate β_0 and β_1 using OLS. Bourguignon and Ferreira (2005)

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[25] Note that the structural effect can also be expressed as $\Delta^y_S = \left(y_{i1} - y^b_{i0}\right)$. In the notation associated with equation (8), linearity and rank preservation imply that y^b_{i0} corresponds to the counterfactual outcome obtained by replacing the outcome structure $\varphi_1(\cdot)$ with $\varphi_0(\cdot)$. In other words, y^b_{i0} is the same as $y_{0|t=1}$. This suggests that the Juhn-Murphy-Pierce (1993) decomposition can be performed in two steps as follows. Start with the overall difference $\Delta^y_O = (y_{i1} - y_{i0})$. Then add to and subtract from this difference the counterfactual outcome y^b_{i0}. This yields a twofold decomposition of the overall difference into the composition and structural effects. Finally add to and subtract from the structural effect the counterfactual outcome y^c_{i0}. This step leads to the final threefold decomposition. Ignorability guarantees that the composition effect is due solely to changes in the distribution of observables.

explain that empirical implementation of a *rank-preserving transformation* is complicated by the fact that both samples do not necessarily have the same number of observations. However, if one is willing to assume that both distributions are the same up to some proportional transformation, then the rank-preserving transformation can be approximated by multiplying residuals in the base period by the ratio of the standard deviation in the end period to the one in the initial period.

Fortin, Lemieux and (2011) point out that assuming constant returns to unobservable and homoskedasticity allows one to write the unobserved component of the outcome as $v_t = \sigma_t \varepsilon$. Homoskedasticity implies that the conditional variance of ε is constant (and can be normalized to 1). Equation (30) can therefore be written as follows.

$$y_t = x_t \beta_t + \sigma_t \varepsilon,\ t = 0, 1 \qquad (32)$$

As it turns out, this is the version of the model used by Juhn, Murphy and Pierce (1991) in their study of the evolution of the wage differential between Blacks and Whites in the U.S.A. In that context, the standard deviation of the residuals in the wage equation stands for both within-group inequality in the wage distribution and the price of unobserved skills (Yun 2009).

The outcome model specified in equation (32) has also been used to study gender pay gap. In that context, t=1 is taken to represent males while t=0 stands for females, and the wage regime for males is considered the non-discriminatory one. The counterfactual used in the decomposition is the outcome female workers would have experienced if they had been paid like their male counterparts. Care must be taken when applying this version of the model to decompose differences in mean outcome using OLS since the OLS residuals sum up to zero. To see this, consider the following expression of standard Oaxaca-Blinder decomposition that explicitly shows the residuals.

$$\Delta_O^{\mu} = [E(x_1) - E(x_0)]\beta_1 + E(x_0)(\beta_1 - \beta_0) + [E(\epsilon_1) - E(\varepsilon_0)]\sigma_1 + E(\varepsilon_0)(\sigma_1 - \sigma_0) \qquad (33)$$

The terms associated with the unobservables in the right hand side of equation (33) will disappears if the decomposition is based on OLS applied to each equation separately.

To get around this issue, Juhn, Murphy and Pierce (1991) assume that the returns to observable characteristics are the same for both groups and apply OLS to only one group, and construct an auxiliary equation for the other group. In the context of gender wage gap studies, OLS is applied to the equation for males only. The equation for female workers is constructed as follows: $y_0 = x_0\beta_1 + v_0$. The implied decomposition is:

$$\Delta_O^{\mu} = [E(x_1) - E(x_0)]\beta_1 - E(v_0) = [E(x_1) - E(x_0)]\beta_1 - \sigma_1 E(\eta_0) \qquad (34)$$

where $\eta_0 = \frac{v_0}{\sigma_1}$. The above expression is computed on the basis of the sample analogs of the parameters of interest. The first term in the twofold decomposition presented in (34) represents the predicted gap while the second stands for the residual gap. As Yun (2009) points out, the residual gap is equal to the structural effect in the standard Oaxaca-Blinder decomposition. Yet, this structural effect represents returns to observable characteristics. It is therefore hard to see how the Juhn, Murphy and Pierce (1991) procedure helps identify the contribution of unobservables. Yun (2009) proposes instead the decomposition defined by equation (33), under the assumption that the expected value of unobservable terms is not equal to zero. However, that author does not provide an implementation procedure corresponding to this situation.

4. Behavioral Responses to Changes in the Socioeconomic Environment

A key limitation of both the basic Oaxaca-Blinder decomposition and its generalization along the lines discussed in sections 2 and 3 is that they do not account for changes in the behavior of agents in response to changes in their socioeconomic environment which may be due to shocks or policy reform. Given the maintained hypothesis that the living standard of an individual in a given society depends crucially on what he decides to do with his assets (innate and external) subject to the opportunities offered by society, this section focuses on ways of modeling agents' behavior to account for their reaction to changes in their socioeconomic environment. Standard economic theory explains behavior in terms of the principles of *optimization* and *market interaction*. Modeling behavior entails the specification of the following elements (Varian 1984): (i) actions that a socioeconomic agent can undertake, (ii) the constraints she faces, and (iii) the objective function used to evaluate feasible actions. The assumption that the agent seeks to maximize the objective function subject to constraints implies that outcome variables used to represent the consequences of behavior can be expressed as functions of parameters of the socioeconomic environment, embedded in the constraints facing the agent. We consider two basic modeling frameworks, namely the consumption-leisure choice paradigm and the Roy (1951) model of choice and consequences. While both frameworks stem from the optimization principle, the consumption-leisure choice model is a straight forward interpretation of standard consumer choice theory in cases where the choice set is continuous. The Roy model applies to discrete choice problems.

4.1. The Consumption-Leisure Choice Paradigm

We first describe the structure of this framework and then explain how counterfactual decomposition can be performed on the basis of an empirical model. Known limitations of the framework will also be noted along the way.

Structure

Standard neoclassical labor supply models are framed within the logic of individual choice between consumption goods, c, and leisure, ℓ. This framework helps one establish the determinants of labor supply and the conditions for participation in the labor market. Given a wage rate, w, and non-wage income y, the consumer is assumed to maximize a utility function of consumption and leisure subject to the *full income* constraint based on time endowment. Let ℓ_{max} be the time endowment representing the maximum amount of leisure the agent can enjoy. The length of time worked is equal to: $h = \ell_{max} - \ell$. Formally, the agent's problem can be stated as:$\max_{c,\ell}[u(c,\ell)|c \leq wh + y]$. Equivalently, we have the following representation in terms of the indirect utility function.

$$v(p,w) = \max_{c,\ell}[u(c,\ell)|c + w\ell \leq m \equiv w\ell_{max} + y] \quad (35)$$

where p stands for the price of consumption normalized to unity in the budget constraint and m_0 defines full income. This formulation of the problem suggests that the wage rate is viewed both as the price and the opportunity cost of leisure. Two basic income sources determine the choice set along with the price of consumption and the wage rate. These are activities within and without the labor market.

The solution to the above optimization problem will lead to an observed supply of labor that is greater or equal to zero indicating respectively participation and nonparticipation in the labor market. Whether or not the agent decides to participate in the labor market depends on a comparison of the wage rate with the trade-off she is willing to make between consumption and leisure as characterized by her utility function (Cahuc and Zylberberg 2004). Within this standard model, it is assumed that both consumption and leisure are normal goods and that the agent is willing to sacrifice less and less consumption for each extra unit of leisure. The rate at which the agent is willing to trade leisure for consumption is indicated by the marginal rate of substitution defined by: $MRS_{\ell c} = \frac{\partial u/\partial \ell}{\partial u/\partial c}$. The agent is willing to supply labor as long as the marginal rate of substitution is less than the market wage rate. At the optimum, the marginal rate of substitution is equal to the wage rate[26]. Thus the wage rate at which the agent finds it optimal to supply zero hours of work is known as the *shadow* or the *reservation wage* (Deaton and Muellbaouer 1980, Cahuc and Zylberberg 2004). At this point, the agent has no labor income, so that the maximum value of consumption is equal to y_0. Since the time endowment is fixed, the reservation wage is a function mainly of non-labor income. When leisure is a normal good, any policy that leads to an increase in non-wage income will increase the reservation wage and will thus have a disincentive effect on the participation decision. The reservation wage thus determines the conditions of participation in the labor market.

As an outcome of the optimization process described by (35), the Marshallian demand for leisure and the corresponding Marshallian supply of labor depend on two parameters, the wage rate, w, and full income m_0 which also depends on the wage rate (the opportunity cost of leisure). It is important to note how a change in the wage (possibly induced by policy or other factors in the socioeconomic environment) might affect the supply of labor. To be specific, consider an increase in the wage rate. This increase will affect the demand for leisure through conventional *substitution* and *income effects*. Since leisure is assumed to be a normal good, these two effects combine to reduce the demand for leisure hence to increase the supply of labor. In addition to these effects, an increase in the wage rate increases the value of full or potential income. This would induce an increase in leisure or a decrease in labor supply. Cahuc and Zylberberg (2004) refer to the conventional income effect as the "indirect" income effect, and call "direct" income effect the one linked to potential income. The overall effect of a change in the wage rate is

[26] To see clearly what is involved, consider the case where $MRS_{\ell c}<w$. This implies that the marginal utility of leisure is less than the leisure value of the marginal utility of consumption. It is therefore desirable for the agent to increase consumption relative to leisure. Assuming decreasing marginal utility, increasing consumption will progressively bring down the marginal value of consumption until equilibrium is reached indicating the optimal combination of consumption and leisure.

therefore ambiguous. It depends on which group of effects dominates. In any case, the shape of the labor supply curve is determined by the interaction among these effects.

Bourguignon and Ferreira (2003) expand the basic framework to include individual characteristics and the net tax system. It is assumed that, an economic agent with characteristics z chooses between consumption and leisure (or labor supply) so as to maximize utility subject to a budget constraint that explicitly incorporate the net tax system[27]. Formally, the problem is stated as follows.

$$v(p,w;z,\varepsilon) = \max_{c,\ell} u(c,\ell;z;\ \beta,\ \varepsilon)|c \leq y + wh + nt(wh,h,y;\ z;\ \xi), h \geq 0 \quad (36)$$

In this expression, nt(·) represents the tax-benefit schedule, a function of the agent's characteristics, his labor income (wh), his exogenous non-labor income (y), and possibly the level of labor supply, h. The parameters defining the tax-benefit system such as tax rates or the means-testing of benefits are represented by ξ. There are also parameters such as β and ε characterizing preferences along with agent's characteristics, z.

Individuals are members of families or households. One would expect that the family or the household will have a considerable influence on decisions made by its members, including labor supply decisions. It is therefore instructive to consider how the basic model discussed so far has been adapted to account for the potential influence of the structure of the household to which an individual belongs. A straightforward extension of the neoclassical model to household-level supply of labor has taken the form of the so-called *unitary model* (Blundell and MaCurdy 1999). A key characterization of this model is the assumption of "*income pooling*" which implies that the level of household consumption is determined by the common pool of resources available for various household members. The household can therefore be viewed as single agent with its own utility that depends on total consumption and individual members' consumption of leisure. Within this framework, individual labor supply functions depend on the household full income, the price of consumption and the wage rates.

The analysis of household choices can also be framed within the logic of the *collective model*. This approach insists on the principle that choices made by a household must reflect the preferences of its members and is consistent with the assumption that individual decisions made

[27] This constraint may also include the fixed cost of employment as a function of individual characteristics. This cost may include cost incurred for child-care while at work. Accounting for this dimension makes it possible to simulate the implications of policy designed to compensate individuals or households for child care (Creedy and Duncan 2002).

within the household are Pareto efficient so that there are no opportunities for mutually advantageous allocation (Cahuc and Zylberberg 2004). In the context of this framework, decisions made by individual i are an outcome of the following mathematical program.

$$v_i(p, w_i) = \max_{c_i,\ell_i}[u(c_i, \ell_i)|c_i + w_i\ell_i \leq m_i \equiv w_i\ell_0 + s_i] \tag{37}$$

where s_i is interpreted as a "sharing rule" determining the share that each household member gets out of total non-wage income of the household. This variable is a function of the wage rate faced by the individual and her contribution in non-wage income. Interestingly, this formulation offers the possibility of inferring the consumption of individual household members which is not necessarily observable. Given the interdependence among choices of different members of the household, this approach makes it easier to understand why certain members of the household may choose to specialize in household production while others may supply their labor to relevant markets. This interdependence also suggests that variations in individual's income affect not only his labor supply decisions, but those of the other members of the household as well.

One key feature limiting the applicability of the basic model of consumption and leisure choice to the study of behavioral responses to shocks and policies is the assumption of a linear budget constraint. More generally, Deaton and Muellbauer (1980) argue that the simple neoclassical labor supply model has limited usefulness for policy analysis because it does not account for issues related to aggregation, participation decisions, constraints on hours worked and unemployment. With respect to the number of hours worked, Salanié (2003) notes that individual choice may be constrained by existing labor market regulations. Furthermore, part time work may not be an outcome of free choice, but rather a reflection of the difficulty in finding full time employment. It is therefore important to consider the participation decision.

Counterfactual Decomposition

In principle, the solution of the program specified in (36) for individual i (or a household depending on the unit of analysis) leads to a simultaneous equation system describing the demand for consumption and leisure. The demand for leisure can be translated into a labor supply function. Given relevant data, the model composed of the consumption and labor supply can be estimated simultaneously first using baseline data, then using end period data. Counterfactual simulations can then be conducted by switching parameters and variables between the two estimated models in order to identify the effect of the factor that was changed while holding all the others constant. This, again, is an application of the *ceteris paribus* identification strategy.

Bourguignon and Ferreira (2003), in the context of ex ante evaluation of policy reforms, propose a recursive approach that can be adapted to the types of decompositions considered here. These authors note that the program stated in equation (36) leads, for individual i, to a nonlinear labor supply function of the form:

$$h_i = h(z_i, w_i, y_i; \beta, \varepsilon_i, \xi). \tag{38}$$

where the ε_i are the *idiosyncratic* preference terms analogous to the random disturbance terms in standard regression analysis. Given a reliable cross-section data set, estimation procedures seek to minimize the role of these idiosyncratic terms in explaining cross-sectional variations in labor supply. This will produce a set of estimates $\hat{\beta}$ for preference parameters and $\widehat{\varepsilon_\iota}$ for the idiosyncratic preference terms. The fact that the preference parameters have no subscript here implies that they are assumed common to all agents. The estimated version of equation (38) can be written as follows.

$$h_i = h(z_i, w_i, y_i; \hat{\beta}, \widehat{\varepsilon_\iota}, \xi). \tag{39}$$

Given a baseline estimate of the labor supply function, it is possible to simulate, for instance, the implications of alternative tax-benefit system for labor supply and consumption by changing relevant components of ξ, and comparing the new results to the base line.

To fix ideas, let $h_{it} = h(z_{it}, w_{it}, y_{it}; \widehat{\beta_t}, \widehat{\varepsilon_{\iota t}}, \xi_t)$ be the labor supply observed for individual i at time t=0, 1. Consider an experiment where the tax system prevailing in the end period is imposed on agents in the baseline period, *ceteris paribus*. Let $h_{is} = h(z_{i0}, w_{i0}, y_{i0}; \widehat{\beta_0}, \widehat{\varepsilon_{\iota 0}}, \xi_1)$ be the corresponding labor supply. The change in labor supply due to this change is equal to the following.

$$h_{is} - h_{i0} = h\big(z_{i0}, w_{i0}, y_{i0}; \widehat{\beta_0}, \widehat{\varepsilon_{\iota 0}}, \xi_1\big) - h(z_{i0}, w_{i0}, y_{i0}; \widehat{\beta_0}, \widehat{\varepsilon_{\iota 0}}, \xi_0) \tag{40}$$

Under the simplifying assumption that consumption is equal to disposable income[28], the budget constraint implies that the corresponding change in consumption is equal to:

$$c_{is} - c_{i0} = w_{i0}(h_{is} - h_{i0}) + \Delta nt(\cdot) \tag{41}$$

where $\Delta nt(\cdot) = nt(wh_{is}, h_{is}, y_{i0};\ z_{i0};\ \xi_1) - nt(wh_{i0}, h_{i0}, y_{i0};\ z_{i0};\ \xi_0)$. These changes in the indicator of economic welfare can then be used to compute the changes in poverty (or any other

[28] No generality is lost here since one can always compute consumption as a fraction of disposable income.

social evaluation function) corresponding to the underlying *ceteris paribus* variation in the tax-benefit system. While we used the tax benefit system to illustrate the principle, it is obvious that the same principle applies to any of the arguments of the labor supply function, be it an observable variable or an estimate of a parameter or term associated with preferences. The case of idiosyncratic terms deserves special attention and we will come back to it in the next subsection. Also, the discussion assumes that the individual is the unit of analysis. However, the general principle carries over to the unitary model of household behavior.

There are serious difficulties associated with the estimation of labor supply models based on this classical approach. These difficulties stem mainly from the assumed continuity of the decision variable, hours of work, coupled with a lack of restrictions on this variable. Salanié (2003) explains, in a simple framework, a structural estimation procedure applicable to this basic model. The optimization process underlying the labor supply function has two possible outcomes. The agent will supply zero hours if the reservation wage is greater than the market wage, otherwise he will supply labor up to the point where the marginal rate of substitution between consumption and leisure is equal to the wage rate. This equality defines a latent labor supply function of the net wage rate, disposable non-labor income and an error term. Since wages are observed only for people who work, a Tobit model of labor supply is estimated jointly with a wage equation that includes individual characteristics (observable and idiosyncratic).

When a full tax-benefit system is included in the budget constraint, the marginal tax rate is increasing and the budget constraint is convex, one can define a virtual wage rate on the basis of the marginal tax rate and a virtual income based on this virtual wage in order to derived the labor supply from an optimization problem that maximizes utility subject to the virtual budget constraint. The fact that the virtual wage and income are endogenous means that we should consider instrumental variable or maximum likelihood estimation methods. When the tax-benefit system leads to non-convex budget constraints, one can no longer rely on first-order conditions to find the optimum. The common approach is to consider discrete levels of labor supply such as h=0, 10, 20, 30, 40 (hours per week) and compare utility values over the choice set to find the maximum. Creedy and Duncan (2002) review a series of estimation procedures that have been used in the empirical literature to deal with these difficulties. They conclude that the structural discrete choice approach offers a more promising method. We briefly outline that approach next, within the logic of the Roy (1951) model of choice and consequences.

4.2. The Roy Model of Choice and Consequences

This section reviews the structure of the basic Roy model along with its interpretation in the context of modeling the determinants of the living standard as represented by household consumption expenditure. It also discusses key considerations in simulating counterfactual distributions underlying any decomposition exercise.

Structure and interpretations

Heckman and Honoré (1990) explain that the original Roy (1951) model was designed for the study of occupational choice and its implications for the distribution of earnings in an economy where agents are endowed with different sets of *occupation-specific skills*. In that economy, income-maximizing agents can freely choose to work only in one of two activities, fishing and hunting, on the basis of their productivity in each. Thus, an agent with a given skill endowment will choose to work in the sector where her potential income is higher. There are no investment opportunities for the augmentation of sector-specific skills nor are there costs associated with changing sectors. These authors also show that self-selection implies a lower level of inequality in earnings compared to a benchmark case where workers are randomly assigned to jobs. The fact that occupational choice has significant implications for the distribution of earnings makes the Roy model a relevant framework for the analysis of behavioral responses by agents to changes in their socioeconomic environment.

Heckman and Sedlacek (1985) discuss an extension of the basic framework by including nonmarket activity as an option in the choice set facing socioeconomic agents who are now assumed to *maximize utility* instead of income. The utility of participating in each of the sectors depends on both sector-specific attributes such as the wage rate or employment risk and job status, and individual characteristics. The fact that we observe only sectoral choices and not the underlying utility function means that it is possible to identify only parameters associated with differences in utility across sectors. These authors also consider the contribution of self-selection to income inequality and find that, in this general model, self-selection can increase both between and within sector inequality compared to a random allocation of workers to sectors.

At the most fundamental level, the Roy model is characterized by two components, namely, a selection mechanism and the associated potential outcomes. These outcomes are possible consequences of the choice made through the selection mechanism. The extended version of the Roy model is consistent with discrete choice models to the extent that utility maximizing agents face a discrete choice set. Train (2009) characterizes a discrete choice model in terms of two fundamental elements: the *choice set* and the *decision process* (or the decision rule). The choice set is the collection of alternatives from which the decision maker chooses one. This set must be *exhaustive* in the sense that it must include all possible alternatives, the latter being *mutually exclusive* from the perspective of the decision maker. Finally, the number of alternatives must be *finite*. In the case of discrete models of labor supply, for instance, the choice set can be represented by a few options such as not working, working part-time and working full time.

Just as in the case of the *consumption-leisure paradigm*, the decision process assumes utility-maximizing behavior. It is therefore assumed that the decision maker chooses the alternative that provides the greatest net benefit or utility. Let $u_{hj}, j = 1, 2, \dots m$ be the utility agent h gets from alternative j. The decision rule implies that alternative k is chosen by the agent if and only if $u_{hk} > u_{hj}\ \forall j \neq k$. This decision making process is usually framed within the logic of the random utility model where utility has two parts. The first, known as the *representative utility*, is a function of some observable characteristics of the decision maker and of the alternatives (Train 2009). The second component is a set of non-observable random factors. Formally, the utility function is written as;$u_{hj} = v_{hj} + \varepsilon_{hj}$ where v is the representative utility and ε represents the unobserved portion of utility that is treated as random. Now, the statement that alternative k is chosen if and only if $u_{hk} > u_{hj}\ \forall j \neq k$ can be equivalently expressed as: k is chosen if and only if $(\varepsilon_{hj} - \varepsilon_{hk}) < (v_{hk} - v_{hj})\ \forall j \neq k$. Because of the uncertainty implied by the random part of the utility function, one can only make probabilistic statements about the decision maker's choice. The probability that option k is chosen by agent h is defined by the following expression[29]:

$$P_{hk} = Pr\left[(\varepsilon_{hj} - \varepsilon_{hk}) < (v_{hk} - v_{hj})\ \forall j \neq k\right] \qquad (42)$$

[29] The expression of this probability can be made more precise by considering an indicator function for the decision rule. The indicator is equal to 1 when option k is chosen and 0 otherwise. The probability that the agent chooses option k is then equal to the expected value of this indicator function over all possible values of the unobserved factors. In other words, $P_{hk} = \int I\left[(\varepsilon_{hj} - \varepsilon_{hk}) < (v_{hk} - v_{hj})\ \forall j \neq k\right] f(\varepsilon_h) d\varepsilon_h$. This is in fact a multidimensional integral over the joint density of the random vector the elements of which represent unobserved factors associated with each alternative. This probability can be interpreted as the proportion of people within the population who face the same observable utility as h for each alternative and choose k (Train 2009).

The type of discrete choice model derived from the above probability statement is determined by the assumptions made about the distribution of the unobserved portion of the utility function. For instance, the common logit model assumes that the random factors are independently and identically distributed (iid) extreme value variables for all options. In other words, each choice is independent from the others[30].

Coulombe and McKay (1996) provide an interesting interpretation of the Roy model that is consistent with our maintained hypothesis that the living standard of an individual is a pay-off from her participation in the life of society. Using the household as the unit of analysis, these authors argue that the living standard of a household depends fundamentally on the socioeconomic group to which it belongs (or their economic activity status). To frame this view within the logic of the Roy model, the authors further argue that one needs to explain the selection mechanism leading to the observed socioeconomic group, and conditional on that choice, the determinants of the living standard in that group. This logic leads to a two-equation model, one representing the selection mechanism and the second modeling the living standard conditional on the choice of a particular socioeconomic group.

Modeling the selection mechanism boils down to modeling the probability defined in equation (42). Consistent with the random utility framework underlying this expression, and assuming that the random elements are generated independently by an extreme value distribution, the multinomial logit model can be used to explain the probability of choosing an option. Formally, we express that probability as:

$$P_{hk} = \frac{\exp{(z_{hk}\gamma_k)}}{\left[1+\sum_{j=2}^{m}\exp{(z_{hj}\gamma_j)}\right]} \tag{43}$$

where z_{hj} is the set of relevant explanatory variables and m is the total number of socioeconomic groups. The probability defined in (43) is essentially the propensity score.

The specification of the explanatory variables requires a good understanding of the determinants of the choice of a socioeconomic group. Autor (2009) explains that there are three

[30] The generalized extreme value model (GEV) allows correlation among unobserved factors. The standard multinomial logit assumes that the random factors are iid with a double exponential distribution. The probit model assumes that the random factors are jointly distributed normal variables. Train (2009) points out that the identification of discrete choice models relies heavily on the fact that only differences in utility matter and the scale of utility is irrelevant. Hence, only parameters that capture differences across alternatives are identifiable and therefore estimable. This also implies that characteristics of the decision maker that do not vary across alternatives will have no effect unless they are specified in a way that induces differences in utility over alternatives. Glick and Sahn (2006) handle this problem by indexing the coefficients of sociodemographic variables in the representative utility function.

technological factors that affect this choice in the context of the general Roy model, namely: (i) the distribution of skills and abilities, (ii) the correlations among these skills in the population and (iii) the technologies for applying these skills. Coulombe and McKay (1996) make a similar point in a case study of Mauritania. They define socioeconomic groups in terms of the income-generating opportunities available to households and their members. In particular, they consider four mutually exclusive and exhaustive groups of households: (i) households working predominantly as employees (whether in the public or private sector), (ii) those engaged mostly in self-employment in agriculture, (iii) those engaged mainly in non-farm self-employment, and (iv) those not in the labor force. In essence, socioeconomic groups are determined on the basis of the main economic activity of the household or the main source of income.

As to the determinants of the choice of socioeconomic groups, these authors argue that the choice depend on variables (such as education, wage or profit rates) that affect relative returns from economic activities as well as on consumption preferences. In particular, they make the point that the extent to which household members choose self-employment over wage employment or to stay out of the labor market depends on the interaction between total household labor supply within and outside the household (a consumption decision) and it total labor demand (a production decision) for both household members and hired labor. In other words, the socioeconomic classification of the household reflects both consumption parameters such as the demographic composition of the household and the characteristics of the head of household, and production parameters relevant to self-employment such as fixed inputs and variable costs.

Equation (43) models the selection mechanism. We need an outcome equation to complete the model within the logic of the Roy framework. Following Coulombe and McKay (1996), we let y_{hk} stand for the log of per capita expenditure for household h in socioeconomic group k, and η_{hk} a random disturbance. The outcome equation associated with equation (43) can be written by analogy to the standard Mincer equation (in labor economics) as follows.

$$y_{hk} = x_h\beta_k + \eta_{hk} \tag{44}$$

Equations (43) and (44) constitute a system designed to explain living standard at the household level. In their case study, Coulombe and McKay (1996) distinguish two categories of determinants, demographic factors that are relevant to all households regardless of the group they belong to. These demographic variables include household size, household composition, and characteristics of the economic head of the household (e.g. education level, marital status, gender and ethnicity). Group specific factors include those affecting the level of total household income. For those engaged in wage employment, such factors would include level of education, sector of employment and numbers of hours worked in a year to account for seasonal work. Given that such variables are difficult to measure at the household level (the unit of analysis), one could define and measure these variables only for the economic head of household or adopt some form of aggregation over household members. Naturally, this would entail some loss of the heterogeneity found at the individual level. In the case of agricultural self-employment, specific factors include such things as land size and quality, tenure status, use of fertilizer, insecticides, hired labor, access to extension services and commercialization. Similar considerations apply to non-agricultural self-employment. For households outside the labor market, possible sources of livelihood include assets holding, borrowing, public and private transfers.

Another important consideration here is the classification of variables as exogenous or endogenous. This classification hinges on the time horizon chosen. Coulombe and McKay (1996) note, for instance, that in the long run the living standard can affect demographic variables such as household size and composition. But in the short run it is reasonable to think of the direction of influence as running from demographic variables to the living standard. In their study, these authors adopt a short to medium time frame so that most of the variables listed above are considered exogenous with respect to the model described by equation (43) and (44).

Bourguignon, Ferreira and Leite (2008) show how to expand this framework to model changes in education and household demographics. In the extended framework, socioeconomic group, per capita consumption, education and household composition are endogenous. Variations in education and in household composition are modeled within the discrete choice framework portrayed by equation (43). In that particular application, the demand for education is modeled on the basis of six alternatives: 0 years of schooling; 1-4; 5-6; 7-8; 9-12; and 13 and more. The highest level of education is the excluded category. The variables considered as purely exogenous by these authors include: number of adults in the households, the region of residence, age, race and gender. For household demographics, the options are: 0, 1, 2, 3, 4, 5 and more children. The last category is omitted in the estimation. Note that education is an explanatory variable in the demographic multilogit model. Leite, Sanchez and Laderchi (2009) apply this extended framework to analyze the evolution of urban inequality in Ethiopia. They too focus on the household as the unit of analysis and use per capita household expenditure as the outcome variable.

Cogneau and Robilliard (2008) use the extended Roy model to study the implications of targeted poverty reduction policies in Madagascar[31]. While using the household as the unit of analysis and considering consumption as the ultimate welfare indicator, these authors model first the income-generating process at the level of individual members of the household, and then link consumption to household income. The choice set facing individuals of working age (15 years and older) includes three alternatives: *family work*, *self-employment* and *wage work*. Household composition and location are exogenously given. For self-employment and wage work, the potential earnings of an individual are equal to a task price times a given idiosyncratic amount of efficient labor. Efficient labor is assumed to be a function of some observable characteristics (such as age, experience and location) and unobservable skills. Family work is rewarded by a reservation wage that is a function of individual and household characteristics[32]. In the absence of labor market segmentation, the simple selection rule of the basic Roy model would base sector choice on a comparison of the reservation wage, and potential wages in the other two sectors. To account for labor market segmentation, Cogneau and Robilliard (2008) define a segmentation variable in terms of the relative cost of entry between self-employment and wage work, and adjust the selection rule accordingly.

For the purpose of policy evaluation, these authors embed the occupational choice model into a broader microeconomic module that includes the demand system for consumption goods. To keep things simple, they assume that consumption or saving decisions are separable from labor supply decisions. They also assume a fixed common saving rate of 0.052 so that aggregate consumption is equal to the implied propensity to consume times disposable income. The latter is the sum of farm profits, labor income, earnings from self-employment, and nonlabor income such as capital income and transfers. Total consumption is allocated to three composite goods (agricultural, informal and

[31] These include (i) a direct subsidy on agricultural production prices, (ii) a workfare program and (iii) a uniform untargeted per capita transfer program.

[32] For agricultural households, earnings are computed on the basis of a reduced farm profit function (based on Cobb-Douglas technology) that includes self-consumption and accounts for hired labor. For family members participating in farm work, the reservation wage (a measure of the value of family work) is assumed to depend on their contribution to farm profits.

formal) according to budget shares derived from available data. The three policies considered have the potential of inducing large macroeconomic effects, because their cost represents about 5 percent of gross domestic product (GDP). To account for this, the authors link the micro module to a small three-sector (agriculture, informal and formal) computable general equilibrium model. The integrated framework makes it possible to consider the macroeconomic impact of the policy options along with their impact on inequality and poverty. Adding a general equilibrium model removes a key limitation of the decomposition methods discussed up to this point. These methods rely on either a purely statistical or a microeconomic model of behavior that cannot account for general equilibrium effects.

Simulating Counterfactual Distributions

The simulation of counterfactual distributions needed for the decomposition of distributional changes proceeds in the same manner as in the case of the simultaneous model of consumption and labor supply discussed above. We need to estimate some version of the Roy system (composed of a selection equation and an outcome equation) for the initial and end periods. Counterfactual distributions can then be simulated by switching parameters and variables between these two estimated models one element at a time holding all the other factors constant.

In general, parameters of sample selection models can be estimated with two-stage methods or the maximum likelihood approach. We focus here on two-step procedures that are also known as *control function methods* or *generalized residual methods* (Todd 2008). As noted earlier, the selection mechanism is usually modeled within a random utility framework and identifying assumptions are based on *functional form restrictions* or *exclusion restrictions* (analogous to the instrumental variable approach). In particular, the control function approach seeks to model conditional expectations of potential outcomes (given observable characteristics and occupational or socioeconomic status) in a way that relates unobservable determinants of outcomes to the observables, including the choice of a socioeconomic group. This is consistent with the view that the underlying endogeneity problem is due to omitted variables. The control functions represent the omitted variables. To fix ideas, consider a two-sector Roy model. Let d be a dummy variable indicating the sector of activity. Thus we let d=1 for sector 1 (e.g. fishing) and d=2 for sector two (hunting)[33] with the corresponding potential outcomes y_1 and y_2.

Given observable characteristics x, z and socioeconomic status, d, the conditional expectations of these potential outcomes can be written as:

$$E(y_1|x,z,d=1) = \beta_1(x) + E(\eta_1|x,z,d=1) \quad (45)$$

Similarly,

$$E(y_2|x,z,d=2) = \beta_2(x) + E(\eta_2|x,z,d=2) \quad (46)$$

Heckman and Navarro-Lozano (2004) explain that if one can model $E(\eta_1|x,z,d=1)$ and $E(\eta_2|x,z,d=2)$ and find a way to cause these functions to vary independently of $\beta_1(x)$ and $\beta_2(x)$, then one can identify $\beta_1(x)$ and $\beta_2(x)$ up to constant terms. This is another manifestation of the separability condition discussed earlier in section 3.

Consistent with the random utility framework underlying the selection model, we assume that d=2 if some underlying index $\mathrm{u} = h(x,z) + \varepsilon$ is greater than zero. Furthermore, if we assume that $(\eta_1, \varepsilon) \perp (x,z)$, then we have:

$$E(\eta_1|x,z,d=1) = E\big(\eta_1 \big| \varepsilon < -h(x,z)\big) = K_1(P(x,z)) \quad (47)$$

where $P(x,z) = P(d=2|x,z)$ is the propensity score. Similarly, assuming that $(\eta_2, \varepsilon) \perp (x,z)$ yields the following relation.

$$E(\eta_2|x,z,d=2) = E\big(\eta_2 \big| \varepsilon > -h(x,z)\big) = K_2(P(x,z)) \quad (48)$$

This approach is consistent with the view that the underlying endogeneity problem is due to omitted variables. *The control functions represent the omitted variables*. The key assumption in this framework that allows us to express each control function $K_j(\cdot)$, j=1, 2 solely as a function of P(x, z) is the assumption that the observed individual characteristics are independent of the unobservable determinants of selection and outcomes (a form of ignorability) . This assumption, along with the rest of imposed restrictions, isolates a

[33] We avoid using the traditional coding (0, 1) because we reserve it for the base and end periods respectively.

source of identifying variation in selection which helps to determine the parameters of interest. The assumption is formally stated as follows:

$$(\eta_1, \eta_2, \varepsilon) \perp (x, z) \tag{49}$$

Under these assumptions, the conditional expectations for the potential outcome are equal to the following.

$$E(y_1|x, z, d = 1) = \beta_1(x) + K_1(P(x, z)) \tag{50}$$

Similarly,

$$E(y_2|x, z, d = 2) = \beta_2(x) + K_2(P(x, z)) \tag{51}$$

If we can use z to vary each control function holding x fixed, we can identify $\beta_1(x)$ and $\beta_2(x)$. We note that these control functions are designed to control for selection bias in the estimation of the relevant parameters.

In the context of Heckman (1976, 1979) two-stage estimator, the control function is known as the inverse Mills ratio. Hall (2002) points out the following drawbacks: (1) The conventional standard error estimates are inconsistent, (2) the method does not impose the constraint that the absolute value of the correlation coefficient be less than one, (3) since the normality assumption is required for consistency, the estimator is no more robust than the full maximum likelihood approach that also requires normality. Lee (1983) proposes an alternative way of estimating the inverse Mills ratio, particularly when one does not assume normality for the random error in the selection equation. This involves a general transformation to normality as follows. Let q_τ be the quantiles associated with the predicted probabilities from the first stage of the process. The transformation computes these quantiles by inverting the cumulative standard normal distribution applied to the predicted probabilities. This is essentially an imputation procedure analogous to the one discussed in section 3.

Once the model has been estimated, counterfactual decompositions are performed following the same logic as in the case of the consumption-leisure model. Building on the statistical approaches discussed in section 3, Bourguignon, Ferreira and Leite (2008) propose the combination of parametric and nonparametric techniques in constructing the desired counterfactuals. To see clearly what is involved, recall that the density function characterizing the joint distribution of the outcomes and covariates can be written as a product of the two underlying density functions, one characterizing the conditional distribution of outcomes given the covariates and the other the joint distribution of covariates. Earlier, we expressed this relation as: $J_t(y,x) = g_t(y|x)h_t(x), t = 0,1$. As noted earlier, this factorization suggests that counterfactual distributions can be obtained by combining the conditional outcome distribution from one period (e.g. initial period) with the joint distribution of covariates from the other period (e.g. the end period). An example of this type of combinations would be the following:

$$J_{g_0}^{h_1}(y,x) = g_0(y|x)h_1(x) \tag{52}$$

A key distinction between the methods discussed in section 3 and those reviewed in this section is that methods in section 3 are based on statistical models of the conditional outcome distribution while the methods discussed here rely on economic modeling of this conditional distribution. Thus, equations (43) and (44) characterizing the basic Roy model must be seen as modeling the conditional outcome distribution $g_t(y|x)$. The method of Bourguignon, Ferreira and Leite (2008) consists in using the parametric approach in generating counterfactuals for the conditional outcome distribution and non-parametric sample reweighing techniques to construct counterfactuals for the joint distribution of exogenous covariates. They argue that the parametric approach for the conditional distribution has the advantage of providing a clear economic interpretation of the parameter estimates along with great flexibility in exchanging parameters from one period to another (i.e. from one state of the world to another).

To see how this works in the context of the Roy framework, use the estimated model to write the approximation to the conditional outcome distribution as follows:

$$y_t = s(x_t, z_t;\ \widehat{\gamma_t}, \widehat{\beta_t},\ \widehat{\varepsilon_t},\ \ \widehat{\eta_t}\ ) \tag{53}$$

Thus a change in the conditional outcome distribution due to a *ceteris paribus* change in the parameters of the multinomial logit model of selection can be computed easily as follows.

$$\Delta y = s\big(x_0, z_0;\ \widehat{\gamma_1}, \widehat{\beta_0},\ \widehat{\varepsilon_0},\ \ \widehat{\eta_0}\ \big) - s(x_0, z_0;\ \widehat{\gamma_0}, \widehat{\beta_0},\ \widehat{\varepsilon_0},\ \ \widehat{\eta_0}\ ) \tag{54}$$

When a counterfactual requires a normalization of exogenous covariates for both periods, we can simply apply the DiNardo, Fortin and Lemieux (1996) approach described in section 2. The handling of the residuals in this process requires some care. In the case of the residuals associated with the outcome equations, for instance, one can resort to the *rank-preserving transformation* described in section 3.

5. Concluding Remarks

The design and implementation of effective strategies for poverty reduction require a relevant and reliable analytical input. The bedrock of this analytical input is certainly a rich and reliable data set (both qualitative and quantitative) to be used in poverty measurement and analysis. In this context, there is a need for a sound understanding of the fundamental factors that account for observed variations in poverty either across space or over time. This paper reviews some of the basic decomposition methods that are commonly used to identify sources of variation in poverty outcomes, both at the macro and micro levels. The paper focuses on micro approaches because macro methods fail to account for the heterogeneity of the factors that drive the observed changes in aggregate poverty.

The decomposition of changes in poverty can be viewed as an exercise in social impact evaluation understood as an assessment of changes in *individual* and *social* outcomes attributable to socioeconomic shocks or policy implementation. All decomposition methods reviewed in this paper obey the same logic of counterfactual decomposition organized around the following terms: domain, outcome model, scope, identification and estimation. The *domain* represents the type of distributional changes a method seeks to decompose (e.g. changes in poverty over time or across space).

The *outcome model* links the outcome of interest to its determining factors. As shown in the appendix, some macro-decomposition methods link variations in poverty to changes in the mean and relative inequality characterizing the underlying distribution of living standards. Others exploit the structure of additively decomposable measures to decompose such variations into an intrasectoral effect and an effect due to population shifts. Outcome models that underlie micro-decomposition methods are consistent with the view that the living standard of an individual is a pay-off from her participation in the life of society, and a function of *endowments, behavior* and the *circumstances* that determine the *returns* to these endowments from any social transaction. These elements define the potential scope of micro-decomposition methods. In general, the *scope* of a decomposition method is the set of explanatory factors the method tries to uncover by decomposition. The specification of an outcome model thus determines the potential scope of the corresponding decomposition method.

The micro-decomposition methods reviewed in this paper fall into two basic categories, namely *statistical* and *structura*l. All seek to model the joint distribution of the outcome variable and its determining factors. This joint distribution can be factorized into a product of the conditional outcome distribution and the marginal distribution of exogenous (independent) variables. Statistical methods rely uniquely on statistical principles to model the conditional outcome distribution while structural methods rely on both economics and statistics to model this object. In particular, the structural methods considered here use utility maximization in a partial equilibrium setting to characterize individual behavior and social interaction. Statistical methods therefore are purely *descriptive,* while structural ones are considered *predictive*.

Identification concerns the assumptions needed to recover, in a meaningful way, the factors of interest at the population level. These assumptions involve both the functional form of the outcome model and the joint distribution of factors that determine the outcome. While macro- and micro-decomposition methods differ in their scope, they share the same fundamental *identification strategy* based on the notion of *ceteris paribus* variation. The implementation of this idea entails the comparison of an observed outcome distribution with a *counterfactual* obtained by changing one factor at a time while holding all the other factors constant. A key counterfactual used in the identification of endowment and price effects is the outcome distribution that would have prevailed in one state of the world had individual characteristics been rewarded according to the system applicable in the alternative state. The construction of this counterfactual relies critically on ignorability and the absence of general equilibrium effects. When the outcome model is separable in observables and unobservables, one can assume *rank preservation* to further split the price effect into a component due to observables and another due to unobservable factors.

Estimation involves the computation of the relevant parameters on the basis of sample data. There is a powerful analogy between the decomposition methods reviewed here and treatment effect analysis. Both fields of inquiry rely on the same fundamental identification strategy, and the *structural effect* is known to be equivalent to the *treatment effect on the treated*. This analogy has led to the development of flexible estimation methods for endowment and structural effects. Nonparametric estimation methods, such as inverse probability weighing, allow the analyst to decompose distributional changes without having to assume a functional form for the outcome model. The downside however is the inability to further decompose the structural effect and to account for behavior. Parametric methods are more suitable for these two tasks.

While the analogy between decomposition methods and treatment effect analysis has helped with the development of estimation methods, it does not necessarily confer a *causal interpretation* to decomposition results. As noted by Ferreira (2010), such an interpretation requires the construction of counterfactual outcome distributions that are fully consistent with a general equilibrium of the economy. One way of achieving this consistency is to base decomposition on a full structural model of *behavior* and *social interaction*. Such a model can be built by embedding a behavioral model, e.g. the Roy (1951) model of choice and consequences, in a general equilibrium framework.

Appendix

Identification of Macro Factors

The macro-decomposition methods reviewed in this appendix are designed to reveal aggregate factors that might explain variations in poverty over time or across socioeconomic groups. As noted in the introduction, these macro methods rely on the same identification strategy as the micro methods discussed in the main text. Each decomposition method is characterized mainly by the nature of the factors it seeks to reveal and the structure of the relationship it assumes between the focal object of decomposition and its determining factors. The first group of methods we consider seek to account for changes in poverty in terms of changes in the mean and in relative inequality of the underlying outcome distribution. We refer to the component associated with the mean as the *size effect*. The one associated with relative inequality will be called the *redistribution effect*. The second group of methods we examine here exploit the structure of additively decomposable poverty measures to characterize changes in poverty over time in terms of *intrasectoral effects* and effects stemming from *population shifts*.

I. The Size and Redistribution Effects

There are two methods of decomposing variations in poverty outcome into the size and redistribution effects. The first is a threefold decomposition that identifies a third component as an indicator of the interaction between the two main effects (size and redistribution). The second method, based on the Shapley value, involves no such term. We review both methods along with ways of simulating relevant counterfactual outcomes.

A Threefold Decomposition

Datt and Ravallion (1992) observe that poverty measures may be fully characterized by the poverty line, z, the mean of the distribution of economic welfare, μ, and relative inequality as represented by the Lorenz curve L. When working with real income as an indicator of economic welfare, the poverty line is considered fixed so that we write the overall level of poverty at time t as a function only of mean income and the Lorenz function: $P_t = P(\mu_t, L_t), t = 0, 1$. The overall change in poverty from base period 0 to end period 1 can be written as follows[34].

$$\Delta_O^P = P(\mu_1, L_1) - P(\mu_0, L_0) \tag{A.1}$$

These authors propose a threefold decomposition procedure that allows the analyst to express overall change in poverty in terms of a component linked to change in the mean only, another component associated with change in the Lorenz curve only and a residual which is in fact the *interaction effect*. In particular, the *size effect* is the change in poverty due to a variation in the mean while the Lorenz curve is fixed at some reference level. Similarly, the *redistribution effect* is the change in poverty due to a change in the Lorenz curve while holding the mean at some reference level. In principle, one could choose either the base period or the end period as reference. However, Datt and Ravallion (1992) argue that the base period is a natural choice for the decomposition and conduct their analysis on that basis. Within that framework, the size effect is equal to the following expression.

$$\Delta_\mu^P = P(\mu_1, L_0) - P(\mu_0, L_0) \tag{A.2}$$

Similarly, the redistribution effect is:

$$\Delta_L^P = P(\mu_0, L_1) - P(\mu_0, L_0) \tag{A.3}$$

Note that these two expressions describe counterfactual outcomes. The size effect entails *distribution neutral* growth (the Lorenz curve does not change). The redistribution effect implies that growth is *size neutral* (the mean does not change).

[34] According to the logic of counterfactual decomposition discussed in the text, $P_t = P(\mu_t, L_t)$ is the outcome model underlying this decomposition. It is a model of a social outcome and its expression is equivalent to $P_t = P(F_{y|t})$, in the notation we use in the text. While we are focusing here on poverty outcomes, it is important to note that these decomposition methods apply as well to generic distributional statistics (social outcome indicators), which we can now express as $\theta_t = \theta(\mu_t, L_t)$.

To obtain the Datt-Ravallion decomposition, we can add and subtract these counterfactual outcomes to and from the right hand side of (A.1). Upon rearranging terms we get the following.

$$\Delta_O^P = \Delta_\mu^P + \Delta_L^P + \big[[P(\mu_1, L_1) - P(\mu_1, L_0)] - [P(\mu_0, L_1) - P(\mu_0, L_0)]\big] \qquad \text{(A.4)}$$

The third term in brackets on the right hand side of (A.4) is the residual interpreted as the interaction effect. It is the difference between two ways of computing the redistributive effect depending on whether one fixes the end period mean or the base period mean. In other words, the residual is the difference between the redistribution effect computed on the basis of the end period mean and the same effect evaluated at the initial mean (Datt and Ravallion 1992, Ravallion 2000).

Interestingly, we can rearrange terms within the residual and get the following equivalent expression.

$$\Delta_R^P = [P(\mu_1, L_1) - P(\mu_0, L_1)] - [P(\mu_1, L_0) - P(\mu_0, L_0)] \qquad \text{(A.5)}$$

This expression reveals that the residual is also equal to the difference between the size effect computed on the basis of the end period Lorenz curve and the same effect evaluated at the initial period Lorenz curve (Datt and Ravallion 1992, Ravallion 2000).

The structure of the residual revealed by equations (A.4) and (A.5) led Datt and Ravallion (1992) to interpret this residual as the *interaction effect* between the size and redistribution effects. Indeed, if the size effect depends on the reference Lorenz curve or the redistribution effect on the reference mean, the residual would not equal zero. Thus, as noted by Datt and Ravallion (1992) the interaction term would vanish if the poverty measure is additively separable between μ and L[35]. These authors also point out that the residual would vanish if one took the average of its components over the base and final years. As it turns out, this is precisely the procedure proposed by Shorrocks(1999) based

[35] Ravallion(2000) clarifies this point by noting that, in general, if a variable v is a function of two variables x and y and if this function is additively separable in x and y, then we can write: v=g(x) + h(y). In these circumstances, the change in v when x changes holding y constant depends only on the initial and final values of x. Without this additive separability, we should expect the variation in v to depend on the particular value of y chosen.

on the Shapely value. Kakwani (2000) proposes the same procedure, but does not refer to the Shapley value.

A Twofold Decomposition Based on the Shapley Value

The Shapley value provides a formula for dividing a joint cost or a jointly produced output on the basis of a fair assessment of individual contributions to the formation of total cost or the production of a surplus. Thus, it can be viewed as an interpretation of the *reward* principle of distributive justice (Moulin 2003)[36]. Formally, the Shapley value is a solution to a cooperative game. The problem of the *commons* is used often to explain the nature of such games. A commons is a technology that is jointly owned and operated by a group of agents. Young(1994) provides the following definition of the Shapley value in the case of cost sharing.

"Given a cost-sharing game on a fixed set of players, let the players join the cooperative enterprise one at a time in some predetermined order. As each player joins, the number of players to be served increases. The player's *cost contribution* is his net addition to cost when he joins, that is, the incremental cost of adding him to the group of players who have already joined. The Shapley value of a player is his average cost contribution over all possible orderings of the players."

To see how the above principle translates into a decomposition procedure, consider a distributional statistic such as the overall level of poverty or inequality. Let it be a function of m contributory factors which together account for the value of the indicator. The decomposition approach proposed by Shorrocks (1999) is based on the marginal effect on the value of the indicator resulting from eliminating sequentially each of the contributory factors and computing the corresponding marginal change in the statistic.

[36] Moulin (2003) argues that the concept of *fairness* can be interpreted in terms of four basic ideas: *exogenous rights*, *compensation*, *reward* and *fitness*. An exogenous right is a normative postulate that dictates how a resource must be distributed among claimants. Equal treatment of equals is an example of such a postulate. In general, exogenous rights set claims to resources independently of the use of such resources and of the contribution to their production while compensation and reward relate fairness to individual characteristics relevant to the use or production of the resources under consideration. The compensation principle advocates giving extra resources to people who find themselves in unfortunate circumstances for which they cannot be held (morally) responsible. The reward principle bases allocation on individual behavior to the extent that it affects the overall burden or advantage under distribution. Finally, according to the fitness principle, resources must go to the person who can make the best use of them.

The method then assigns to each factor the average of its marginal contributions in all possible elimination sequences.

The Shapley decomposition rule respects the following restrictions: (1) *Symmetry* or *anonymity*, meaning the contribution assigned to any factor should not depend on its label or the way it is listed; (2) the result must be an exact and *additive decomposition*; and (3) the contribution of each factor is taken to be equal to its (first round) *marginal impact*.

To see how this logic applies to the decomposition of change in poverty over time into a size and a redistribution effect with no residual, rewrite equation (A.1) in the following form:

$$\Delta_O^P = H(\Delta\mu, \Delta L) \qquad \text{(A.6)}$$

In other words the overall change in poverty is fully determined by two contributory factors[37], namely the change in the mean of the distribution $\Delta\mu = \mu_1 - \mu_0$ and the change in the Lorenz curve $\Delta L = L_1 - L_0$. As can be seen in the case of the Datt-Ravallion method, the value of any effect (size or redistribution) depends on the chosen period of reference. This *path-dependence* violates the anonymity constraint that the Shapley method must respect. We therefore need to consider all possible sequences of elimination and the associated marginal contributions that must be averaged in the end. In this simple case, we have only two possible sequences: either we eliminate the size factor first by setting $\Delta\mu = 0$, then the redistribution factor by setting $\Delta L = 0$, or we start with the redistribution factor to end with the size factor.

Consider eliminating the size factor first. Then the change in poverty would be:

$$H(0, \Delta L) = P(\mu_0, L_1) - P(\mu_0, L_0) \qquad \text{(A.7)}$$

It is clear from expression (A.7) that this change is attributed to the redistribution factor. Given that the decomposition must be exhaustive and additive, the corresponding size effect is equal to the following.

[37] Thus $H(0,0) = 0$.

$$\Delta_{\mu}^{P} = \Delta_{O}^{P} - H(0, \Delta L) = P(\mu_1, L_1) - P(\mu_0, L_1) \tag{A.8}$$

For the alternative sequence, we eliminate the redistribution factor first so that the change in poverty becomes:

$$H(\Delta\mu, 0) = P(\mu_1, L_0) - P(\mu_0, L_0) \tag{A.9}$$

This is the left over contribution of the size factor. The corresponding contribution of the redistribution factor is:

$$\Delta_{L}^{P} = \Delta_{O}^{P} - H(\Delta\mu, 0) = P(\mu_1, L_1) - P(\mu_1, L_0) \tag{A.10}$$

The Shapley contribution of the size factor to change in poverty is equal to the average (over the two possible elimination sequences) of the relevant marginal contributions. That is:

$$S_{\mu} = \frac{1}{2}\big[[P(\mu_1, L_1) - P(\mu_0, L_1)] + [P(\mu_1, L_0) - P(\mu_0, L_0)]\big] \tag{A.11}$$

Similarly, the Shapley contribution of the redistribution factor to change in poverty is equal to:

$$S_{L} = \frac{1}{2}\big[[P(\mu_0, L_1) - P(\mu_0, L_0)] + [P(\mu_1, L_1) - P(\mu_1, L_0)]\big] \tag{A.12}$$

While our discussion so far has focused on variation in poverty over time, it is important to note that the Shapley method is applicable to the analysis of differences in poverty across space. Kolenikov and Shorrocks (2005) use the Shapley rule to decompose regional differences in poverty in Russia, working both with real and nominal incomes[38]. In the context of spatial decomposition, these authors refer to the size effect as the *income effect*. Furthermore, the base period must now be interpreted as a reference region which could be the whole country or the capital city for instance. An interesting feature of the case study of Russia is the fact that Kolenikov and Shorrocks (2005) present a threefold decomposition of change in poverty in terms of effects associated with changes in nominal income, inequality and poverty line. The latter represents the regional price effect. In that

[38] Essama-Nssah and Bassole (2010) use the same method to analyze regional disparity in Cameroon.

case, relevant poverty measures are written as: $P(\mu, L, z)$ where z stands for the poverty line.

Recall equations (28) and (29) from section 3. These two equations reveal that, for the class of additively separable poverty measures, the change in poverty can be written as a function of the growth incidence curve (GIC) and therefore inherits the decomposability of that curve. Using the *neutral element* for addition, one can split the GIC into one component showing the growth rate of average income and another showing the deviation of each point on the curve from the overall growth rate. Formally, we write:

$$g(y_i) = \gamma + [g(y_i) - \gamma] \tag{A.13}$$

The first component is the rate of growth that would be experienced at every quantile if the growth process were distribution neutral. This is essentially the size effect. It can be shown that the second component is equal to the change in the slope of the Lorenz curve between the base and end period (Ravallion and Chen 2003). Thus, this component measures the redistribution effect. The corresponding Shapley decomposition of change in poverty is:

$$\Delta_O^P = \frac{\gamma}{n}\sum_{i=1}^{n} y_i \psi'(y_i|z) + \frac{1}{n}\sum_{i=1}^{n} y_i \psi'(y_i|z)[g(y_i) - \gamma] \tag{A.14}$$

This decomposition carries over to the elasticity of poverty with respect to the average income. To see this, just normalize expression (A.14) by γP.

Simulating Relevant Counterfactual Outcomes

The implementation of the above decomposition methods requires ways of estimating the underpinning counterfactual distributions. The selection of an estimation approach depends on the structure of the available data.

If one is using household or individual level data, scaling up the initial distribution by a factor equal to the ratio $\frac{\mu_1}{\mu_0}$ produces a counterfactual distribution with the same Lorenz curve as the initial distribution and the same mean as the end period distribution. This is a distribution neutral transformation that leads to the computation of the following counterfactual outcome: $P(\mu_1, L_0)$. Similarly, the computation of $P(\mu_0, L_1)$ entails scaling up the end period distribution by $\frac{\mu_0}{\mu_1}$.

If instead one has only aggregate data, say outcome distribution by quintile or decile, then one can simulate these counterfactuals on the basis of a parameterized Lorenz function (Datt 1998). To see what is involved, we recall the definition and structure of the Lorenz curve and discuss its parameterization based on the general quadratic model. The informational content of a cumulative distribution function (CDF) can be encoded into a Lorenz function. Rank all individuals in ascending order of the outcome variable (e.g. indicator of living standard). Let p stand for the 100p percent of the population with the lowest outcome values, and L(p) the share of the total outcome value going to that segment of the population. The Lorenz curve maps the cumulative proportion p on the horizontal axis against the cumulative share L(p) on the vertical axis for all p in [0, 1].

Let y be the focal variable with density function f(y) and distribution $F(z) = \int_0^z f(y)dy$. The latter represents the proportion, p, of the population with an outcome value less than or equal to z. The corresponding cumulative share is: $L(p) = \int_0^z \frac{y}{\mu} f(y)dy$, where μ is the average outcome. By definition, $dp = f(y)dy$. Therefore, the cumulative share can also be written as $L(p) = \int_0^p \frac{y(q)}{\mu} dq$. This is the Lorenz function. Its first order derivative is equal to $L'(p) = \frac{y(p)}{\mu}$, and the second order derivative is $L''(p) = \frac{1}{\mu f(z)}$ (see Lambert 2001 for details). These two derivatives reveal that we can recover the quantile or the level of the outcome variable from the information contained in the mean and the first order derivative. Similarly we can recover the density function from the mean and the second order derivative.

For the general quadratic Lorenz model, the Lorenz ordinate is given by the following expression (Datt 1998).

$$L(p) = -\frac{1}{2}\left[\beta_2 p + e + (mp^2 + np + e^2)^{\frac{1}{2}}\right] \tag{A.15}$$

The corresponding first order derivative is equal to:

$$L'(p) = -\frac{\beta_2}{2} - \frac{2mp+n}{4\sqrt{(mp^2+np+e^2)}} \qquad \text{(A.16)}$$

And the second order derivative is:

$$L''(p) = \frac{r^2(mp^2+np+e^2)^{-\frac{3}{2}}}{8} \qquad \text{(A.17)}$$

The parameters characterizing the general quadratic Lorenz model are defined as follows: $e = -(\beta_1 + \beta_2 + \beta_3 + 1)$ where β_1, β_2, and β_3 are the coefficients from the regression of $[L(1-L)]$ on $(p^2 - L)$, $L \times (p-1)$ and $(p-L)$ without an intercept. Furthermore, Datt (1998) explains that one must drop the last observation since the chosen functional form forces the curve to go through (1, 1). With this parameterization, one can perform the counterfactual decomposition described in equation (A.4) or the one implied by equations (A.11) and (A.12) by combining the relevant means and Lorenz functions. For instance, the computation of the counterfactual social outcome $P(\mu_1, L_0)$ is based on the following elemental inputs: μ_1, $L'_0(p)$ and $L''_0(p)$. The corresponding counterfactual outcomes are estimated by the following expression:

$$y_{L_0}^{\mu_1}(p) = \mu_1 \times L'_0(p) \qquad \text{(A.18)}$$

The corresponding value of the density function is equal to:

$$f_{L_0}^{\mu_1}(y) = \frac{1}{\mu_1 \times L''_0(p)} \qquad \text{(A.19)}$$

The same logic applies for the estimation of the other counterfactual, $P(\mu_0, L_1)$.

Most distributional statistics can be computed on the basis of the information on the outcome variable y and the corresponding density function. In that context, $f(y)dy$ is interpreted as the proportion of people whose outcome lies in the close interval $[y, dy]$ for an outcome level y and an infinitesimal change dy (Lambert 2001).

Decomposing Differences in the Generalized Lorenz Ordinates

Finally, note that one can use the Shapley method to decompose changes in a generalized Lorenz curve[39] across quantiles into a size effect and a redistribution effect, and infer the poverty implications of these distributional changes (Sotelsek-Salem et al. 2011). The generalized Lorenz curve is equal to the ordinary Lorenz curve defined above multiplied by the mean of the corresponding distribution. Formally, we write:

$$GL(p) = \mu L(p) = \int_0^p y(q)dq\,, \forall p \in [0,1] \tag{A.20}$$

The generalized Lorenz curve is a social evaluation function that respects both the Pareto principle (more is preferred to less) and the Pigou-Dalton transfer principle (more equality in the outcome distribution is preferred to less). On the basis of these value judgments, social welfare will improve as we move from the baseline distribution (t=0) to the posterior distribution (t=1), if the generalized Lorenz curve associated with the posterior distribution lies nowhere below that of the baseline distribution. This condition is known as *second order dominance*.

One can base *pro-poor judgments* on this dominance relation. For instance, Duclos (2009) explains that, within the relative approach to poverty-focused evaluation, a distributional change that benefits (harms) the poor more (less) than the non-poor must be considered pro-poor[40]. We may denote a relative pro-poor standard by (1+ρ) to indicate the change in the living standards society would like the poor to experience given the overall distributional change. If this standard is set to the ratio of the mean of the posterior distribution to that of the baseline distribution, then a pro-poor change should increase the outcomes of the poor in proportion to the change in the overall mean outcome. Second-order pro-poor judgments based on generalized Lorenz dominance can be stated as follows.

$$GL_1(p) - GL_0(p) \geq \rho GL_0(p), \forall p \in [0, F_1((1+\rho)z^{max})] \tag{A.21}$$

[39] Not to be confused with the general quadratic model of the Lorenz curve.

[40] If one chooses an absolute standard of evaluation, then a pro-poor change increases the poor's absolute standard of living.

Where $F_1(\cdot)$ is the posterior distribution function and z^{max} the maximum level of the poverty line. When the relative standard is set to the ratio of the outcome means, then condition (A.21) is equivalent to having the Lorenz curve of the posterior distribution located above that of the baseline distribution over the relevant range of p.

Given the overall judgment represented by (A.21), one might be interested in identifying the contribution of the size effect and that of the redistribution effect into that overall change. The left-hand side of (A.21) can be written as follows.

$$\Delta GL(p) = \mu_1 L_1(p) - \mu_0 L_0(p) \tag{A.22}$$

Following the same logic as in the case of poverty measures, we compute the size effect as:

$$\Delta_\mu GL(p) = \frac{1}{2}[\mu_1 L_1(p) - \mu_0 L_1(p) + \mu_1 L_0(p) - \mu_0 L_0(p)] \tag{A.23}$$

The corresponding redistribution effect is:

$$\Delta_L GL(p) = \frac{1}{2}[\mu_0 L_1(p) - \mu_0 L_0(p) + \mu_1 L_1(p) - \mu_1 L_0(p)] \tag{A.24}$$

Atkinson (1987) establishes a relationship between second order dominance and poverty reduction. In particular, he proves that if $\Delta GL(p) \geq 0$, then we know that posterior outcome distribution has less poverty than the baseline distribution for all poverty lines and all poverty measures defined by equation (27) in the text that respect the transfer principle. This provides us with a basis for tracking the poverty implications of the size effect or the redistribution effect. By definition, $GL_1(p) = GL_0(p) + \Delta GL(p), \forall p$. In other word, the ordinates of the end-period generalized Lorenz curve are obtained by adding the difference in ordinates to the corresponding ordinates of the baseline generalized Lorenz curve. Following Sotelsek-Salem et al. (2011), instead of adding the total difference, we add only the redistribution effect to obtain the following intermediate generalized Lorenz curve.

$$GL_A(p) = GL_0(p) + \Delta_L GL(p), \forall p \tag{A.25}$$

If this intermediate curve dominates the baseline curve, in order words if the redistribution effect is positive for all p, then the redistribution effect reduces poverty as measured by all members of the additively separable class that respect the transfer principle.

Note that the pro-poor test presented in (A.21) can be restated as follows.

$$GL_1(p) \geq (1+\rho)GL_0(p), \forall p \in [0, F_1((1+\rho)z^{max})] \tag{A.26}$$

This suggests that the redistribution effect will be relatively pro-poor if the following condition holds.

$$GL_A(p) \geq (1+\rho)GL_0(p), \forall p \in [0, F_1((1+\rho)z^{max})] \tag{A.27}$$

In other words, $\frac{\Delta_L GL(p)}{GL_0(p)} \geq \rho, \forall p \in [0, F_1((1+\rho)z^{max})]$. This test for second-order relative pro-poorness entails a comparison of growth rates of the cumulative income of proportions p of the poorest to the standard rate ρ. An equivalent test is based on the so-called three I's of poverty (TIP) curve of Jenkins and Lambert (1997), which is obtained by partially cumulating individual contributions to overall poverty from the poorest individual to the richest, and normalizing.

II. Within-Group and Population Shift Effects

The application of the decomposition methods described above to the class of additively decomposable poverty measures allows the analyst to decompose changes in poverty over time into an effect due to changes in within-group poverty and another due to population shifts. Let the total population of a given country be partitioned exhaustively into m socioeconomic groups. Let w_{kt} be the share of population in group k at time t for t=0, 1, and P_{kt} the level of poverty in that group at the same time. For additively decomposable poverty measures, overall poverty at time t can be written as follows.

$$P_t = \sum_{k=1}^{m} w_{kt} P_{kt} \tag{A.28}$$

The change in aggregate poverty over time can now be written as follows.

$$\Delta_O^P = \sum_{k=1}^{m} [w_{k1}P_{k1} - w_{k0}P_{k0}] \tag{A.29}$$

At this stage, we are interested in accounting for the overall change in poverty Δ_O^P in terms of changes in within group poverty, $\Delta P_k = P_{k1} - P_{k0}$, and the population shifts between groups, $\Delta w_k = w_{k1} - w_{k0}$. We note that the contribution of group k in the change of aggregate poverty is equal to the following expression.

$$C_k = w_{k1}P_{k1} - w_{k0}P_{k0} \tag{A.30}$$

If the population share of this group were fixed at the baseline level, the contribution of this group to overall poverty change would be $w_{k0}\Delta P_k$. We can add and subtract this counterfactual to (A.30), rearrange terms and sum over k. We get the following decomposition presented in Bourguignon and Ferreira (2005).

$$\Delta_O^P = \sum_{k=1}^m w_{k0}\Delta P_k + \sum_{k=1}^m P_{k1}\Delta w_k \tag{A.31}$$

According to (A.31) the overall change in poverty can be split into two components, one representing the contribution of changes in within-group poverty and the other is the contribution of population shifts.

It is possible to further transform this twofold decomposition as follows. Consider the counterfactual where within group poverty does not change. On the basis of equation (A.31), the contribution of group k to change in poverty can be written as:

$$C_k = w_{k0}(P_{k1} - P_{k0}) + P_{k1}(w_{k1} - w_{k0}) \tag{A.32}$$

Fixing group level poverty at the base level reduces this contribution to: $P_{k0}(w_{k1} - w_{k0})$

We can add and subtract this counterfactual to and from equation (A.32), rearrange terms and sum up over k to get the threefold decomposition proposed by Ravallion and Huppi (1991)

$$\Delta_O^P = \sum_{k=1}^m w_{k0}\Delta P_k + \sum_{k=1}^m P_{k0}\Delta w_k + \sum_{k=1}^m \Delta w_k \Delta P_k \tag{A.33}$$

Equation (A.33) shows that change in aggregate poverty over time can be decomposed in three components representing respectively: within-group effects, the effect associated with population shifts and interaction effects.

Shorrocks (1999) also shows that the Shapley principle applies to this situation as well and leads to the following twofold decomposition of change in aggregate poverty over time.

$$\Delta P = \sum_{k=1}^{m} \left(\frac{w_{k0}+w_{k1}}{2}\right) \Delta P_k + \sum_{k=1}^{m} \left(\frac{P_{k0}+P_{k1}}{2}\right) \Delta w_k \tag{A.34}$$

References

Angrist Joshua D. and Pischke Jörn-Steffen. 2009. Mostly Harmless Econometrics: An Empiricist's Companion. Princeton: Princeton University Press.

Atkinson, A. B. 1987. On the Measurement of Poverty. Econometrica, Vol. 55, No.4: 749-764.

Autor David H. 2009. Lecture Note: Self-Selection – The Roy Model. Department of Economics, MIT, http://econ-www.mit.edu/files/4092

Bauer Thomas K. and Sinning Mathias. 2008. An Extension of the Blinder-Oaxaca Decomposition to Nonlinear Models. Advances in Statistical Analysis, Vol. 92, No. 2 :197-206.

Bhaumik Sumon Kumar, Gang Ira N. And Yun Myeong-Su. 2006. A Note on Decomposing Differences in Poverty Incidence Using Regression Estimates: Algorithm and Example. IZA Discussion Paper No. 2262.

Bitler Marianne P., Gelbach Jonah B., and Hoynes Hilary W. 2006. What Mean Impacts Miss: Distributional Effects of Welfare Reform Experiments. American Economic Review, Volume 96, No. 4: 988-1012.

Blinder Alan S. 1973. Wage Discrimination: Reduced Form and Structural Estimates. The Journal of Human Resources, Vol. 8, No.4: 436-455.

Blundell Richard and Macurdy Thomas. 1999. Labor Supply: A Review of Alternative Approaches. In Orley Ashenfelter and David Card (eds), Handbook of Labor Economics , Vol. 3: 1559-1695. New York: Elsevier Science.

Bourguignon François, Ferreira Francisco H. G. and Leite Phillippe G. . 2008. Beyond Oaxaca-Blinder : Accounting for Differences in Households Income Distributions. Journal of Economic Inequality, Vol. 6, No.2, 117-148

Bourguignon François and Spadaro Amedeo. 2006. Microsimulation as a Tool for Evaluating Redistribution Policies. Journal of Economic Inequality, 4: 77-106.

Bourguignon François and Ferreira Francisco H. G. 2005. Decomposing Changes in the Distribution of Household Incomes: Methodological Aspects. In François Bourguignon, Francisco H.G. Ferreira and Nora Lustig (eds), The

Microeconomics of Income Distribution Dynamics in East Asia and Latin America, pp. 17-46 Washington, D.C.: The World Bank.

Bourguignon François, Ferreira Francisco H. G. and Lustig Nora 2005. Introduction. In François Bourguignon, Francisco H.G. Ferreira and Nora Lustig (eds), The Microeconomics of Income Distribution Dynamics in East Asia and Latin America, pp. 17-46 Washington, D.C.: The World Bank

Bourguignon François, Fournier Martin, and Gurgand Marc. 2004. Selection Bias Corrections Based on Multinomial Logit Model: Monte-Carlo Comparisons. DELTA Working Paper No. 2004-20.

Bourguignon François, Fournier M. and Gurgand M. 2001. Fast Development with a Stable Income Distribution : Taiwan, 1979-1994. Review of Income and Wealth, Series 47, No2 : 139-163.

Bourguignon François and Ferreira Francisco H. G. 2003. Ex Ante Evaluation of Policy Reforms Using Behavioral Models. In François Bourguignon and Luiz A. Pereira da Silva (eds), « The Impact of Economic Policies on Poverty and Income Distribution : Evaluation Techniques and Tools. Washington, D.C. : The World Bank.

Cahuc Pierre and Zylberberg André. 2004. Labor Economics. Cambridge (Massachusetts): The MIT Press.

Carneiro Pedro, Hansen Karsten T. and Heckman James J. 2002. Removing the Veil of Ignorance in Assessing the Distributional Impacts of Social Policies. Institute for the Study of Labor (IZA) Discussion Paper No. 453.

Chenery Hollis, Ahluwalia Montek S., Bell C.L. G., Duloy John H. and Jolly Richard. 1974. Redistribution with Growth: Policies to Improve Income Distribution in Developing Countries in the Context of Economic Growth. Oxford: Oxford University Press.

Cogneau Denis and Robilliard Anne-Sophie. 2008. Simulating Targeted Policies with Macro Impact: Poverty Alleviation Policies in Madagascar. In François Bourguignon, Maurizio Bussolo and Luiz A. Pereira da Silva (eds), "The Impact of Macroeconomic Policies on Poverty and Income Distribution: Macro-Micro Evaluation Techniques and Tools". Washington, D.C. : The World Bank.

Cogneau Denis and Robilliard Anne-Sophie. 2007 Growth, Distribution, and Poverty in Madagascar: Learning from a Microsimulation Model in a Genberal Equilibrium Framework. In Amedeo Spadaro (ed), "Microsimulation as a Tool for the Evaluation of Public Policies: Methods and Applications". Bilbao: Fundacion BBVA.

Coulombe Harold and McKay Andrew. 1996. Modeling Determinants of Poverty in Mauritania. World Development, Vol. 24. No.6: 1015-1031.

Creedy John and Duncan Alan. 2002. Behavioural Microsimulation with Labour Supply Responses. Journal of Economic Surveys, Vol. 16, No. 1: 1-40.

Datt Gaurav. 1998. Computational Tools for Poverty Measurement and Analysis. International Food Policy Research Institute (IFPRI), Food Consumption and Nutrition Division (FCND), Discussion Paper No. 50.

Datt Gaurav and Ravallion Martin. 1992. Growth and Redistribution Components of Changes in Poverty Measures: a Decomposition with Applications to Brazil and India in the 1980s. Journal of Development Economics, 38: 275-296.

Deaton, A. 1997. The Analysis of Household Surveys: A Microeconometric Approach to Development Policy. Baltimore: The John Hopkins University Press.

Deaton Angus and Muellbauer John. 1980. Economics and Consumer Behavior. London: Cambridge University Press.

DiNardo John, Fortin Nicole and Lemieux Thomas. 1996. Labor Market Institutions and the Distribution of Wages, 1973-1992 : A Semi-Parametric Approach. Econometrica, Vol, 64, No. 5 : 1001-1044.

Duclos Jean-Yves. 2009. What is "Pro-Poor"? Social Choice and Welfare, **32** : 37-58.

Duncan Alan. 1993. Labour Supply Decisions and Non-Convex Budget Sets: The Case of National Insurance Contribution in U.K. In Alberto Heimler and Daniele Meulders (eds), Empirical Approaches to Fiscal Policy Modelling. (137-162). London : Chapman & Hall.

Elder Todd E., Goddeeris John H. and Haider Steven J. 2010. Unexplained Gaps and Oaxaca-Blinder Decompositions. Labour Economics **17**: 284-290.

Essama-Nssah B. 1997. Impact of Growth and Distribution on Poverty in Madagascar. Review of Income and Wealth, Series 43, No.2 : 239-252.

Essama-Nssah B. and Lambert Peter J. 2011. Influence Functions for Distributional Statistics. Society for the Study of Economic Inequality (ECINEQ), Working Paper No. 2011-236.

Essama-Nssah B. and Bassole Leandre. 2010. A Counterfactual Analysis of the Poverty Impact of Economic Growth in Cameroon. World Bank Policy Research Working Paper No. 5249.

Essama-Nssah, B. and Lambert Peter J. 2009. Measuring Pro-Poorness: a Unifying Approach with New Results. *Review of Income and Wealth,* Series 55, No.3: 752-778.

Essama-Nssah B., Bassole Leandre and Paul Saumik. 2010. Accounting for Heterogeneity in Growth Incidence in Cameroon. World Bank Policy Research Working Paper 5464.

Fairlie Robert W. 2005. An Extension of the Blinder-Oaxaca Decomposition Technique to Logit and Probit Models. Journal of Economic and Social Measurement, Vol. 30, No. 4: 305-316.

Ferreira Francisco H.G.: Distributions in Motion: Economic Growth, Inequality and Poverty Dynamics. 2010. World Bank Policy Research Working Paper No. 5424. The World Bank, Washington, D.C.

Firpo Sergio, Fortin Nicole and Lemieux Thomas. 2009. Unconditional Quantile Regressions. Econometrica, Volume 77, No. 3: 953-973 .

Fortin Nicole, Lemieux Thomas and Firpo Sergio. 2011. Decomposition Methods in Economics. In: Ashenfelter Orley and Card David (eds) Handbook of Labor Economics, Vol. 4A , pp. 1-102. Northolland, Amsterdam.

Foster James, Greer Joel, and Thorbecke Erik. 1984. A Class of Decomposable Poverty Measures. Econometrica, Vol. 52, No. 3:761-766.

Glick Peter and Sahn David. 2006. The Demand for Primary Schooling in Madagascar: Price, Quality and the Choice Between Public and Private Providers. Journal of Development Economics, Vol. 79, No. 1: 118-145.

Grimm Michael. 2005. Educational Policies and Poverty Reduction in Cote d'Ivoire. Journal of Policy Modeling, Vol, 27, No. 2 : 231-247.

Grimm Michael. 2004. A Decomposition of Inequality and Poverty Changes in the Context of Macroeconomic Adjustemnt : A Microsimulation Study for Cote d'Ivoire. In, Growth, Inequality and Poverty : A.F. Shorrocks and R. van der Hoeven (eds) : Prospects for Pro-Poor Economic Development. Oxford : Oxford University Press.

Hall, Bronwyn H. 2002. Notes on Sample Selection Models (mimeo).

Heckman, James J. 1979. Sample Selection Bias as a Specification Error. Econometrica, Vol. 47, No.1: 153-161.

Heckman, James J. 1976. The Common Structure of Statistical Models of Truncation, Sample Selection and Limited Dependent Variables and a Simple Estimator fro such Models. Annals of Economic and Social Measurement, Vol.5, No. 4: 475-492.

Heckman James J. and Navarro-Lozano Salvador. 2004. Using Matching, Instrumental Variables, and Control Functions to Estimate Economic Choice Models. The Review of Economics and Statistics, Vol. 86, No. 1: 30-57.

Heckman James J. and Smith Jeffrey. A. 1995. Assessing the Case of Social Experiments. Journal of Economic Perspectives, Vol. 9, Issue 2:85-110.

Heckman James J. and Honoré Bo E. 1990. The Empirical Content of the Roy Model. Econometrica, Vol. 58. No. 5 : 1121-1149.

Heckman James J. and Sedlacek Guilherme. 1985. Heterogeneity, Aggregation, and Market Wage Functions : An Empirical Model of Self-Selection in the Labor Market. The Journal of Political Economy, Vol. 93, No.6 :1077-1125.

Heinrichs Jutta and Kennedy Peter. 2007. A Computational Trick for Calculating the Blinder-Oaxaca Decomposition and Its Standard Error. Economics Bulletin, Vol. 3, No. 66: 1-7.

Huppi Monika and Ravallion Martin. 1991. The Sectoral Structure of Poverty During an Adjustement Period : Evidence for Indonesia in the Mid-1980s. World Development, Vol. 9, No.12 :1653-1678.

Jann Ben. 2008. A Stata Implementation of the Blinder-Oaxaca Decomposition. ETH (Eidgenössische Technische Hochschule) Zurich Sociology Working Paper No.5 (Swiss Federal Institute of Technology).

Juhn Chinhui, Murphy Kevin M. and Brooks Pierce. 1993. Wage Inequality and the Rise of Returns to Skill. Journal of Political Economy, Vol. 101, No.3 : 410-442.

Juhn Chinhui, Murphy Kevin M. and Brooks Pierce. 1991. Accounting for the Slowdown in Black-White Wage Convergence. In Marvin H. Kosters (ed), Workers and their Wages : Changing Patterns in the United States. Washington DC : American Enterprise Institute.

Kakwani Nanak. 2000. On Measuring Growth and Inequality Components of Poverty with Application to Thailand. Journal of Quantitative Economics, Vol.16, No. 1 :67-79.

Kline Patrick. 2010. Blinder-Oaxaca as a Reweighting Estimator. Department of Economics, University of California at Berkeley, Mimeo.

Kolenikov Stanislav and Shorrocks Anthony. 2005. A Decomposition Analysis of Regional Poverty in Russia. Review of Development Economics, Vol. 9, No. 1 :25-46.

Kraay Aart. 2006. When is Growth Pro-Poor ? Evidence from a Panel of Countries. Journal of Development Economics, **80** : 198-227.

Kunze Astrid. 2008. Gender Wage Gap Studies : Consistency and Decomposition. Empirical Economics, No. 35 : 63-76.

Lambert, Peter J. 2001. The Distribution and Redistribution of Income. Manchester: Manchester University Press.

Lee, Lung-Fei. 1983. Generalized Econometric Models with Selectivity. Econometrica, Vol. 51, No. 2:507-512.

Leite Phillippe G. and Fruttero Anna. 2010. Toolkit on Simulating the Impact of Conditional Cash Transfer Programs www.worldbank.org/safetynets.

Leite Phillippe G., Sanchez Alan, and Laderchi Caterina R. 2009. The Evolution in Urban Inequality in Ethiopia. Washington, DC. : The World Bank (mimeo).

Machado J. and Mata J. 2005. Counterfactual Decomposition of Changes in Wage Distributions Using Quantile Regression. Journal of Applied Econometrics 20 (4): 445-465.

Maddala, G. S. 1983. Limited-Dependent and Qualitative Variables in Econometrics. Cambridge: Cambridge University Press.

Melly Blaise. 2005. Decomposition of Differences in Distribution Using Quantile Regression. Labour Economics, 12: 577-590.

Milgrom Paul. 2004. Putting Auction Theory to Work. Cambridge: Cambridge University Press.

Moulin, Hervé. 2003. Fair Division and Collective Welfare. Cambridge, Massachusetts: The MIT Press.

Neuman Shoshana and Oaxaca Ronald L. 2004. Wage Decompositions with Selectivity-Corrected Wage Equations: A Methodological Note. Journal of Economic Inequality, **2**: 3-10.

Nguyen Binh T., Albrecht James W., Vroman Susan B. and Westbrook M. Daniel. 2007. A Quantile Regression Decomposition of Urban-Rural Inequality in Vietnam. Journal of Development Economics 83: 466-490.

Nielsen Skyt Helena. 1998. Discrimination and Detailed Decomposition in a Logit Model. Economics Letters, 61: 115-120.

Oaxaca Ronald L. 1973. Male-Female Wage Differentials in Urban Labor Markets. International Economic Review, Vol. 14, No.3: 693-709.

Rapoport Anatol. 1999. Two-Person Game Theory. Mineola (New York): Dover Publications.

Ravallion Martin. 2001. Growth, Inequality and Poverty: Looking Beyond Averages. World Development, Vol 29: No. 11: 1803-1815.

Ravallion Martin. 2000. On Decomposing Changes in Poverty into “Growth” and “Redistribution” Components . Journal of Quantitative Economics, Vol. 16, No. 1: 105-118.

Ravallion Martin and Chen Shaohua. 2003. Measuring Pro-Poor Growth. Economic Letters 78, 93-99.

Ravallion Martin and Huppi Monika. 1991. Measuring Changes in Poverty: A Methodological Case Study of Indonesia during an Adjustment Period. The World Bank Economic Review, Vol.5, No5:57-82.

Rosenbaum Paul and Rubin Donald. 1983. The Central Role of the Propensity Score in Observational Studies for Causal Effects. Biometrika, 1983, Vol, 70, No. 1: 41-55.

Rothe Christoph. 2010. Decomposing Counterfactual Distributions. Toulouse School of Economics, mimeo.

Roy A. 1951. Some Thoughts on the Distribution of Earnings. Oxford Economic Papers, **3**: 135-146.

Salanié Bernard. 2003. The Economics of Taxation. Cambridge (Massachusetts): The MIT Press.

Schmertmann, C. P. 1994. Selectivity Bias Correction Methods in Polychotomous Sample Selection Models. Journal of Econometrics, Vol. 60:101-132.

Shorrocks A. F. 1999. Decomposition Procedures for Distributional Analysis: A Unified Framework Based on Shapley Value. University of Essex and Institute for Fiscal Studies, mimeo.

Silverman, B.W. 1986. Density Estimation for Statistics and Data Analysis. London: Chapman and Hall.

Sinning Mathias, Hahn Markus and Bauer Thomas K. 2008. The Blinder-Oaxaca Decomposition for Nonlinear Models. The Stata Journal, Vol. 8, No.4: 480-92.

Sotelsek-Salem Daniel, Ahamdanech-Zarco Ismael, and Bishop John A. 2011. Dominance Testing for "Pro-Poor" Growth with an Application to European Growth. Empirical Economics (forthcoming).

Spadaro Amedeo. 2007. Microsimulation as a Tool for the Evaluation of Public Policies. In Amedeo Spadaro (ed), "Microsimulation as a Tool for the Evaluation of Public Policies: Methods and Applications". Bilbao: Fundacion BBVA.

Todd Petra E. 2008. Evaluating Social Programs with Endogenous Program Placement and Selection of the Treated. In Schultz T.P. and Strauss J. (eds), Handbook of Development Economics, Vol. 4.:3847-3894

Train Kenneth E. 2009. Discrete Choice Methods with Simulation. Cambridge: Cambridge University Press (Second Edition).

Train Kenneth E. and Wilson Wesley W. 2008. Estimation on Stated-Preference Experiments Constructed from Revealed-Preference Choices. Transportation Research Part B, Vol, 42, Issue 3: 191-203.

Varian Hal R. 1984. Microeconomic Analysis. New York: Norton & Company (second edition)

Von Mises R. 1947. On the Asymptotic Distribution of Differentiable Statistical Functions. The Annals of Mathematical Statistics, **18**, 309-348.

Wilcox Rand R.. 2005. Introduction to Robust Estimation and Hypothesis Testing (Second Edition). Elsevier, Amsterdam.

World Bank. 2005. World Development Report 2006: Equity and Development. Washington, D.C. The World Bank and Oxford University Press.

World Bank . 1990. World Development Report 1990: Poverty. Washington, D.C.: The World Bank.

World Bank. 1978. World Development Report, 1978. Washington, D.C.: The World Bank.

Young, H. Peyton. 1994. *Equity: In Theory and Practice*. Princeton: Princeton University Press.

Yun Myeong-Su. 2009. Wage Differentials, Discrimination and Inequality: A Cautionary Note on the Juhn, Murphy and Pierce Decomposition Method. Scottish Journal of Political Economy, Vol. 56, No. 1: 114-122

Yun Myeong-Su. 2005. A Simple Solution to the Identification Problem in Detailed Wage Decompositions. Economic Inquiry, Vol. 43, No. 4: 766-772.

Yun Myeong-Su. 2004. Decomposing Differences in the First Moments. Economics Letters, Vol. 82, No. 2:275-280.